ME3351 - Engineering Mechanics

UNIT I

Statics of Particles

Basics and Vectors

Mechanics

The branch of physical science that deals with the state of rest or state of motion under the action of forces is termed as Mechanics.

Mechanics can be divided into 1. Statics 2. Dynamics

Statics & Dynamics:

Statics analyses the forces on particles or rigid bodies which are at rest, Dynamics deals with bodies which are in motion.

Dynamics can be divided into: 1. Kinematics 2. Kinetics

Kinematics is the study of the relationship between displacement, velocity and acceleration without considering the forces which cause the motion.

Kinetics is the study of motion of the bodies with consideration of forces involved on it.

Basic Concepts:

Space: is used to represent the position of a point - in relation to reference point called Origin.

Time: is used to define the event.

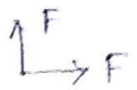
Mass: is used to characterise and compare the bodies. It is used to measure resistance to change the state of rest or motion called Inertia.

Force: is the effort required to change or tends to change the state of rest or uniform motion of a body.

Dimensional analysis is useful.
1. to derive some expressions
2. to find the dimensions of unknown variable.

Sign Convention

1. +ve Force



2. -ve

2. ↻ moment is assumed to be -ve, ↺ moment +ve

Laws of Mechanics

1. Newton's I Law

Everybody continuous in its state of rest or of uniform motion, unless an external force acts on it.

2. Newton's II Law

The rate of change of momentum (MLT^{-1}) of a body is directly proportional to the resultant force acting on it, and takes place in the direction of that force. $\Sigma F = ma$

3. Newton's III Law

For every action, there will be an equal and opposite reaction.

4. Newton's Law of gravitation

Two particles of mass 'm_1' and 'm_2' are attracted towards each other along the line connecting them with a force, whose magnitude is directly proportional to the produce of their masses and inversely proportional to the square of the distance between them.

$$F = \frac{G m_1 m_2}{R^2}$$

where, G = Universal Constant of gravitation

$= 6.67 \times 10^{-11}\ Nm^2/kg^2$.

and R = the distance between the particles.

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5. Parallelogram law

This law states that, when two forces represent the two sides of a parallelogram, then the diagonal will be "Resultant"

6. Principle of transmissibility.

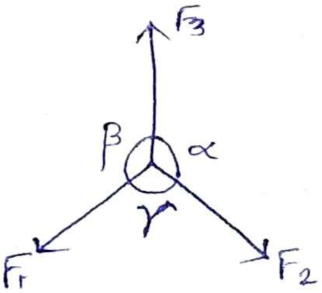
The effect of a force will not be changed, if the point of application is shifted anywhere along the line of action.

7. Lami's theorem:

Lami's theorem states that, if three forces acting on a particle, keep it in equilibrium, then each force is proportional to the sine of the angle between the other two forces and the constant of proportionality is the same.

Mathematically, from fig 1.1 (a)

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$



8. Laws of Triangle:

(i) Sine law:

If a, b, c are the sides of a triangle as shown in fig

fig (a)

and α, β, γ be the angle between the sides then the sine law states that, $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

(ii) Cosine law: If two sides and the angle between the sides are known, then the third side is given by

$$a^2 = b^2 + c^2 - 2bc\cos\theta$$

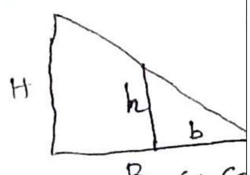


fig. (c)

In a right angle triangle, $\sin\theta = \frac{\text{Opposite Side}}{\text{Hypotenuse}}$

$$\cos\theta = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}$$

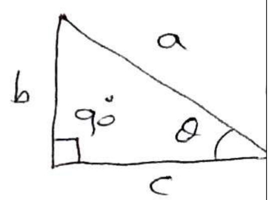


fig (b)

(iii) The Pythagorous theorem says,

$$a^2 = b^2 + c^2 \quad \text{[fig. b]}$$

(iv) Law of similar triangles $\frac{H}{a} = \frac{h}{b}$ [fig. c]

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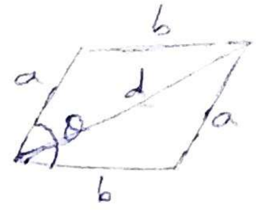
9. Law of Parallelogram.

If a & b are the two sides of a parallelogram, then the diagonal will be given by,

$$d^2 = a^2 + b^2 + 2ab\cos\theta$$



Where, θ is the angle between the two sides as shown in fig.

Scalars and Vectors:

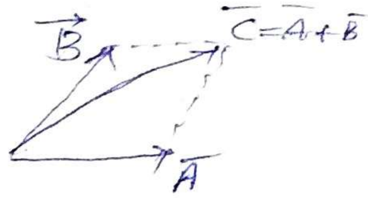
Scalars: A quantity is said to be scalar, if it is completely defined by its magnitude alone.

Vectors: A quantity is said to be vector, if it is completely defined by its magnitude & direction.

Eg. Force, velocity, acceleration, momentum & weight.

Vector Operations:

1. Addition of Vectors



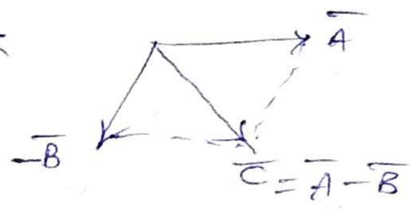
$\bar{A} = a_1 i + b_1 j + c_1 k$; $\bar{B} = a_2 i + b_2 j + c_2 k$ then,

$$\bar{C} = (a_1 + a_2) i + (b_1 + b_2) j + (c_1 + c_2) k$$

2. Subtraction of Vectors

$\bar{A} = a_1 i + b_1 j + c_1 k$; $\bar{B} = a_2 i + b_2 j + c_2 k$

$$\bar{C} = (a_1 - a_2) i + (b_1 - b_2) j + (c_1 - c_2) k$$



3. Dot product of Vectors:

The dot product of two vectors $\bar{A}$ & $\bar{B}$ is given by

$$\bar{A} \cdot \bar{B} = AB\cos\theta$$

Where, 'θ' is the angle between the two vectors. Dot product is also called as 'Scalar Product of Vectors'.

Statics of Particles

Particle is defined as an object that has no size & shape but that has mass.

Force:

Force is an effort required to change the state of rest or of uniform motion of a body. This force can push, pull & twist of a body.

Characteristics of a force

A force can be characteristics by

i) magnitude of force (F)
ii) Point of application (A)
iii) Line of action (θ)
iv) Sense of force [Pull or Push]

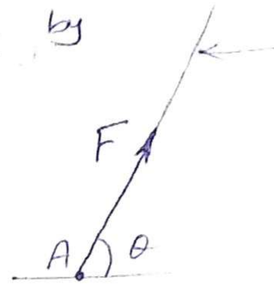
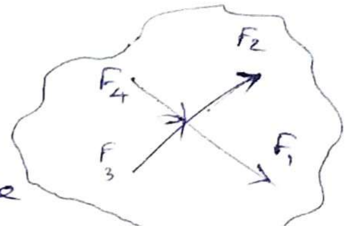


fig. Representation of force

Types of force System:

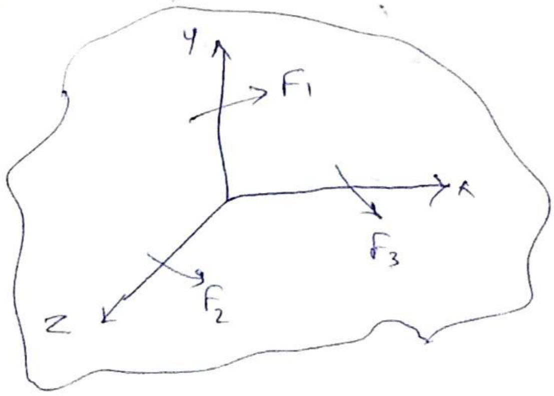
1. Coplanar force System



If all the line of action of forces are lying on a single plane, then these force system is called coplanar force: eg. 2D force system.

2. Non-Coplanar force system or Space force system:

If the lines of action of all forces are not lying on a single plane or 3 Dimentional forces are called Space force system. eg. Tripoid, Transmission tower etc.



Equilibrium of a Particle :-

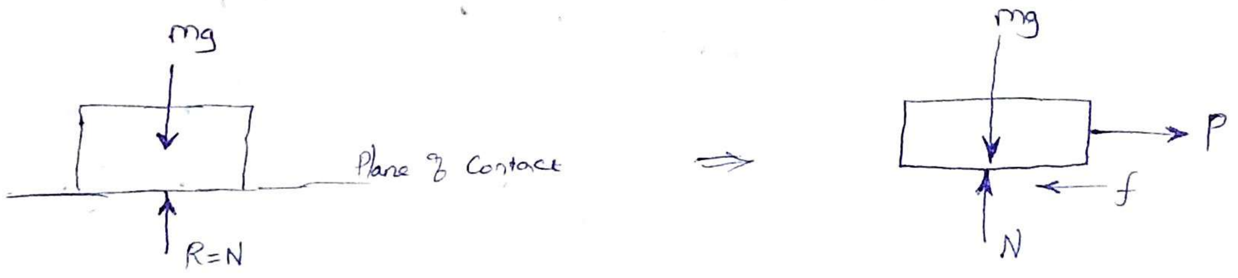
A particle is in equilibrium, provided it is at rest or it has a constant velocity.

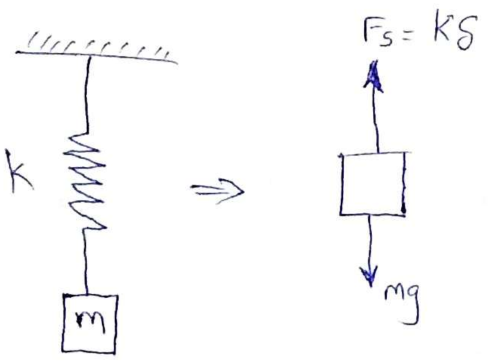
To solve the problem for unknowns, we must draw the free body diagram of the situation.

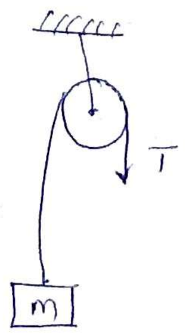
Free body Diagram (FBD):-

It is a diagram which shows all the forces which act on the particle which is being isolated or free from surrounding. To draw the free body diagram, the following forces must be taken into account.

1. Self weight of the body (W),
2. Applied forces (P)
3. Reaction or Normal forces (R or N)
4. Frictional force (f)
5. Spring force [Fs]
6. Cable tension (T)







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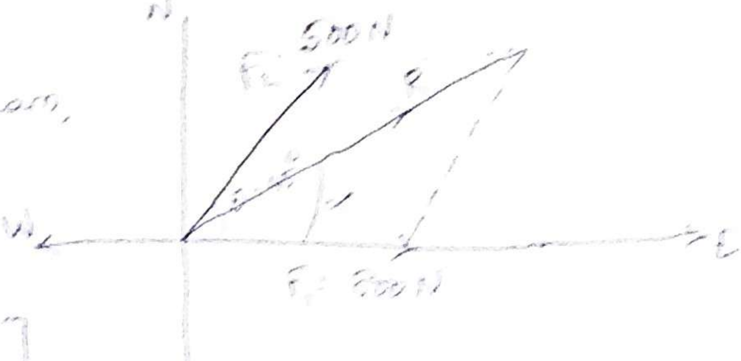
Numerical Problems

1. Find the resultant of an 800 N force acting towards eastern direction and a 500 N force acting towards north eastern directions.

Soln.

(i) Using law of parallelogram,



$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$\boxed{R = 1206.52\ N}$$

$$\alpha\ \theta = \tan^{-1}\left[\frac{F_2\sin\theta}{F_1 + F_2\cos\theta}\right]$$

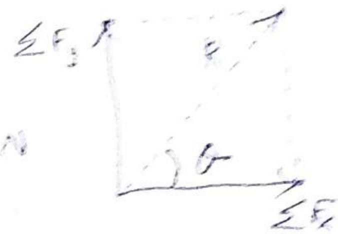
$$\alpha\ \boxed{\theta = 17.04^\circ}$$

If two forces acting simultaneously on a body are represented by the sides of a triangle taken in order, the closing side of triangle taken in the opposite order represents their resultant.

(ii) Summing of Components:

$$\Sigma F_x = F_1 + F_2\cos\theta = 1153.55\ N$$

$$\Sigma F_y = F_2\sin\theta = 353.55\ N$$

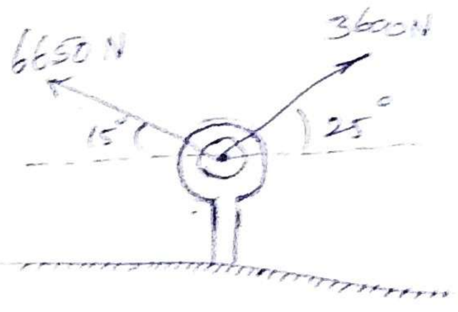


$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = 1206.52\ N,$$

$$R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos(180 - \theta)$$

$$\theta = \tan^{-1}\left[\frac{\Sigma F_y}{\Sigma F_x}\right] = 17.04^\circ,$$

2. Two wires attached to a bolt in a foundation as shown in fig. Determine the pull exerted by the bolt on the foundation.



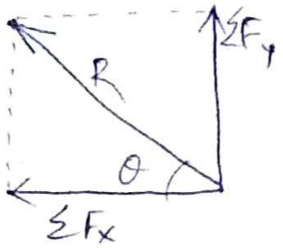
$$\Sigma F_x = 3600\cos25^\circ - 6650\cos15^\circ$$

$$\Sigma F_x = -3160.6987\ N$$

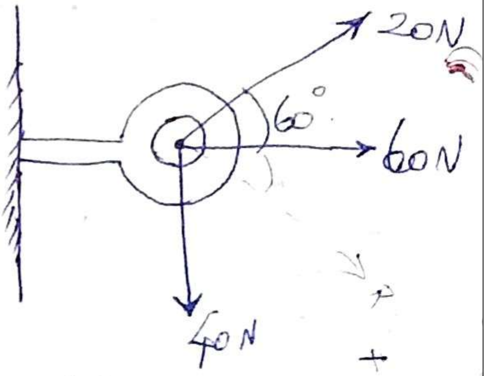
$$\Sigma F_y = 3600\sin25^\circ + 6650\sin15^\circ$$

$$\Sigma F_y = 3242.572\ N$$

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = 4528.167\,N$; $\theta = \tan^{-1}\left[\frac{\Sigma F_y}{\Sigma F_x}\right] = 45.73^\circ$.



3. Three wires exert the tensions indicated on the ring as shown in fig. Assuming as concurrent system, determine the force in a single wire to replace the three wires.



$\Sigma F_x = 60 + 20\cos 60 = 70\,N.$

$\Sigma F_y = -40 + 20\sin 60 = -22.68\,N$

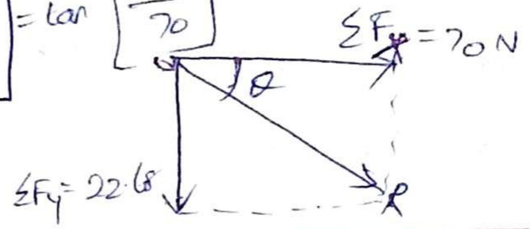
$R = \sqrt{\Sigma F^2 + \Sigma F_y^2}$

$R = \sqrt{70^2 + (-22.68)^2} = 73.582\,N$

~~$\theta = \Sigma F_y$~~

$\theta = \tan^{-1}\left[\frac{\Sigma F_y}{\Sigma F_x}\right] = \tan^{-1}\left[\frac{22.68}{70}\right]$

$\boxed{\theta = 17.952^\circ}$



4. Determine the angle bet two equal forces F, when their resultant is, i) $R = F$; (ii) $R = F/2$

Soln:-

From the law of 11logram the resultant of two forces is gr

i) $R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$

$F^2 = F^2 + F^2 + 2F^2\cos\theta$

$\cancel{F^2} = \cancel{F^2}(2 + 2\cos\theta)$

$1 = 2(1 + \cos\theta)$

$1 + \cos\theta = 1/2$

$\cos\theta = -1/2$

$\boxed{\theta = 120^\circ}$

ii) $\frac{F^2}{4} = F^2 + F^2 + 2F^2\cos\theta$

$\cancel{F^2}/4 = \cancel{F^2}(2 + 2\cos\theta)$

$\frac{1}{4} = 2(1 + \cos\theta)$

$1 + \cos\theta = \frac{1}{8}$

$\cos\theta = \frac{1}{8} - 1$

$\cos\theta = \frac{-7}{8}$

$\boxed{\theta = 151.0^\circ}$

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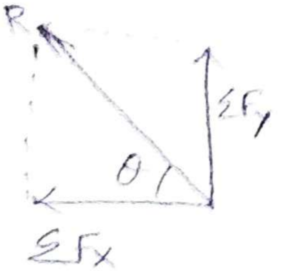
8. Find out the resultant of the system of forces given below

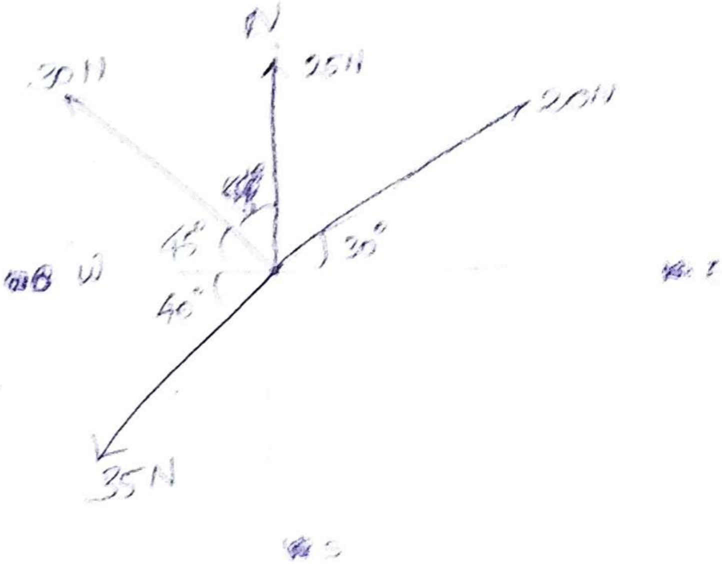
i) 20N inclined at 30° towards north of east

ii) 25 N towards north

iii) 30 N towards north west

iv) 35 N inclined at 40° towards south of west





$$\theta = \tan^{-1}\left[\frac{33.72}{30.69}\right]$$

$$\boxed{\theta = 47.693^\circ}$$

S.No	Force	Angle with x axis	Horizontal Component		Vertical Component	
			Expression	Value	Expression	Value
1.	20 N	30°	+ 20 cos 30		20 sin 30	
2.	25 N	90°	− 25 cos 90		25 sin 90	
3.	30 N	45°	− 30 cos 45		30 sin 45°	
4.	35 N	40°	− 35 cos 40		−35 sin 40°	

$\Sigma F_x =$ −30.69 N

ΣF_y: 33.72

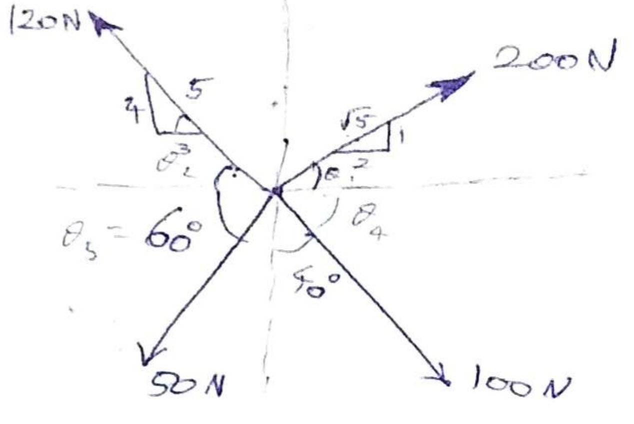
$$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = 45.595\,N$$

10. A system of 4 forces acting on a body as shown in fig. 13
Determine the resultant force and its direction.

$\theta_1 = \tan^{-1}\left[\frac{1}{2}\right] = 26.5651^\circ$

$\theta_2 = \tan^{-1}\left[\frac{4}{3}\right] = 53.1301^\circ$

$\theta_3 = 60^\circ$; $\theta_4 = 50^\circ$.

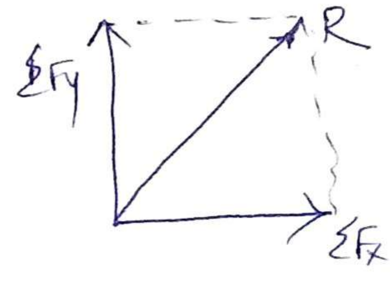


S.No	Force	Angle with x axis	Horizontal Component		Vertical Component	
			Expansion	Value	Expansion	Value
1.	200 N	26.5651	+ 200 cos 26.5651	178.885	+200 sin 26.5651	89.4428
2.	120N	53.1301	−120 cos53.1301	−72.00	+ 120 sin 53.1301	95.9999
3.	50 N	60	−50 cos 60	−25	−50 sin 60	−43.3013
4.	100 N	50	+100 cos 50	64.4278	−100 sin 50	−76.6044
				146.3128		65.5369

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = 160.32\ N$

$\theta = \tan^{-1}\left[\frac{65.5369}{146.3128}\right]$

$$\boxed{\theta = 24.1287^\circ}$$



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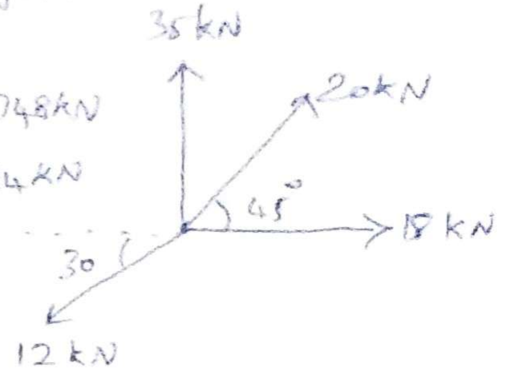
Equilibrant (E)

Equilibrant is a single force having same magnitude with that of the resultant force and acting in the opposite direction, so as to make the body at rest or in equilibrium

1. An unknown force E keep the four coplanar concurrent forces in equilibrium. Find the force.



$\Sigma F_x = 18 + 20\cos 45 - 12\cos 30 = 21.748\,kN$

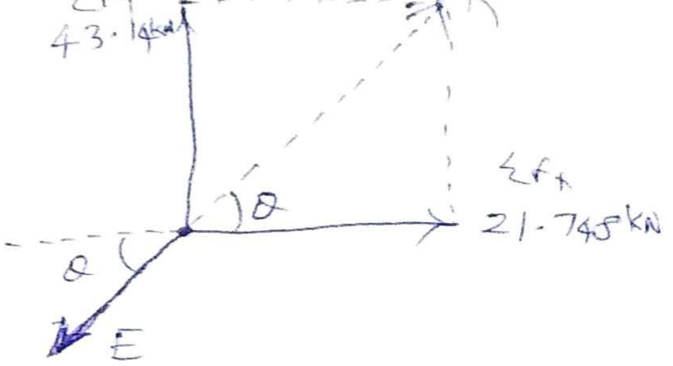
$\Sigma F_y = 35 + 20\sin 45 - 12\sin 30 = 43.14\,kN$

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = 48.3118\,kN$

$\boxed{\theta = 63.246^\circ}$

Equilibrant is a force having same magnitude as that of the resultant and applying in the opp. direction to keep the particle in equilibrium.

E = 48.3118 kN acting at an angle $\theta = 63.246^\circ$ with negative x-axis



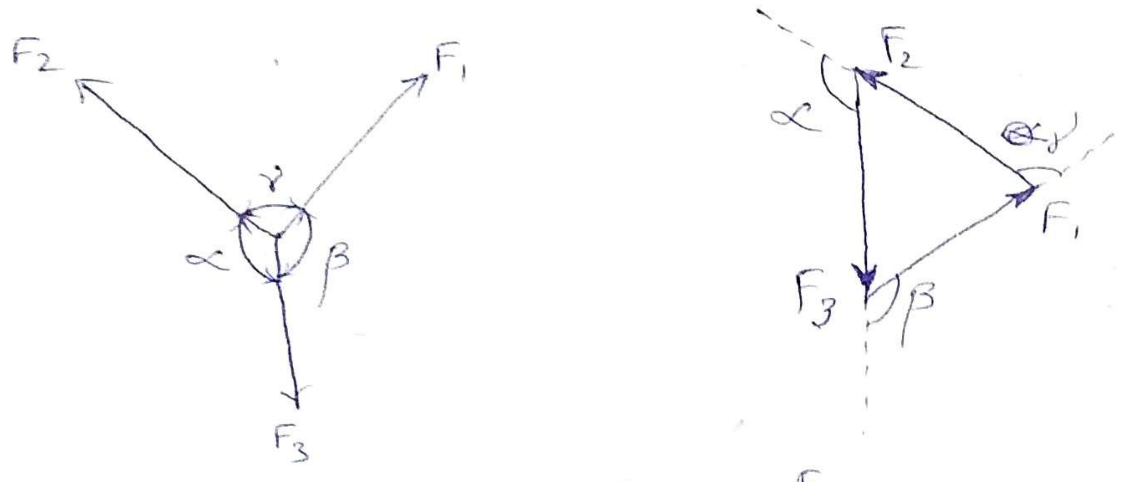
Equilibrium of Particle Subjected to Coplanar forces (2D)

1. A Particle is subjected to two forces:

When a particle 'A' is subjected to two forces and if the particle is in equilibrium, then the two forces will have same magnitude, the same line of action but with opposite sense as shown in fig. (a).

2. A particle is subjected to three forces:

When a particle 'A' is subjected to three forces, and the particle is in equilibrium, it must satisfy the Lami's theorem (Which States that each force is Proportional to the sine of the angle between the other two forces) and they must be concurrent.



$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

3. Equilibrium of a particle, subjected to several forces

When a particle is subjected to several forces, the force must be resolved into horizontal & vertical component. For equilibrium the resultant force $\bar{R} = 0$

$\therefore \Sigma F_x = 0$ .... sum of the horizontal forces $= 0$

$\Sigma F_y = 0$ .... sum of the vertical force $= 0$

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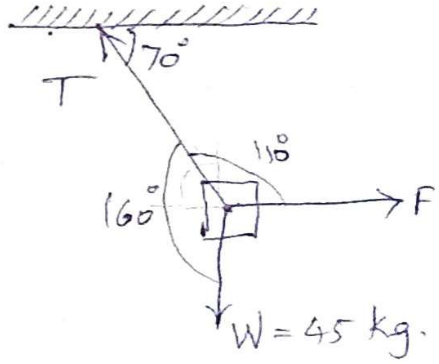
① A mass of 45 kg is suspended by a rope from a ceiling. The mass is pulled of a horizontal force until the rope makes at an angle of 70° with the ceiling. Find the horizontal force and the tension in the rope.

using Lamis Theorem,



$$\frac{F}{\sin 160^\circ} = \frac{(45 \times 9.81)\,N}{\sin 110^\circ} = \frac{T}{\sin 90^\circ}$$

$\underline{F = 160.614\ N}$; $\underline{T = 469.78\ N}$.

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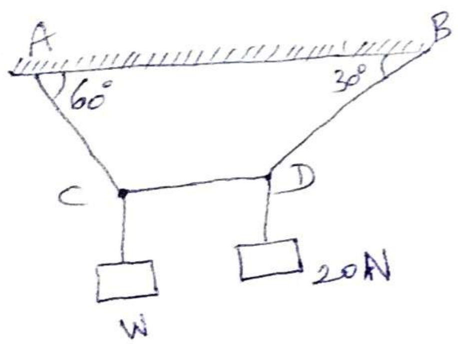
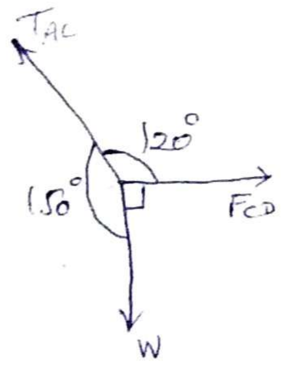
② A cord supported at A & B carries a load of 20 N at D and a load of W at C as shown. Find the value of W so that CD remains horizontal.





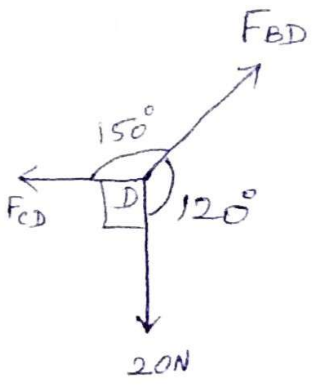


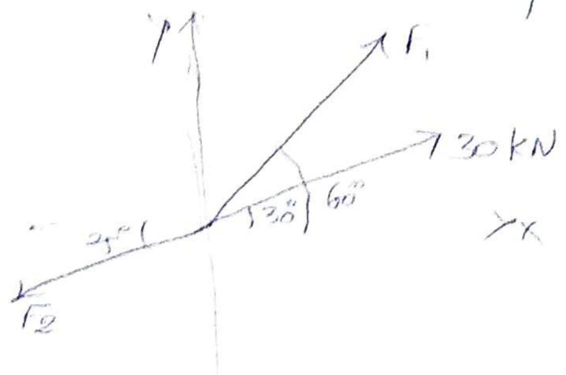
Fig. (b)

$$\frac{30}{\sin 120} = \frac{F_{CB}}{\sin 120} \Rightarrow F_{CB} = 34.641\,N$$

From fig (b)

$$\frac{W}{\sin 120} = \frac{F_{CB}}{\sin 150} \Rightarrow W = 60\,N$$

(5) Determine the forces F_1 & F_2 as the particle is in equilibrium as shown in fig.



$\Sigma F_x = 0$

$F_1 \cos 60 + 30 \cos 30 = F_2 \cos 25$

$F_1 \cos 60 - F_2 \cos 25 = -\cancel{28.6...}\ 5.98076$

$0.5 F_1 - 0.9063 F_2 = -\cancel{28.6...}\ 5.98076$

$\Sigma F_y = 0$

$F_1 \sin 60 + 30 \sin 30 = F_2 \sin 25$

$F_1 \sin 60 - F_2 \sin 25 = -30 \sin 30$

$0.866 F_1 - 0.4226 F_2 = -15$

$F_1 - 0.488 F_2 = -17.321$

$F_1 = -\cancel{...}\ 51.96152 + 1.8126 F_2$ ——— (1)

$F_1 = -17.321 + 0.488 F_2$ ——— (2)

(1) − (2) $\Rightarrow 0 = -\cancel{8.6597}\ 34.6405 + 2.3006 F_2$

~~$2.3006 F_2 = 8.6597$~~ 34.6405

$\boxed{F_2 = 15.057\,kN}$

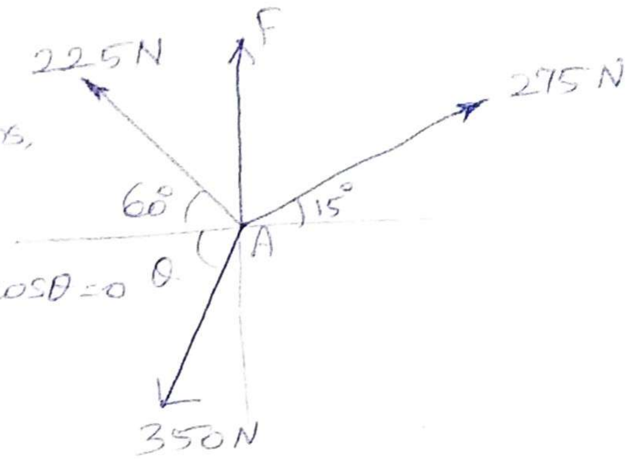
$F_1 = -9.97\,kN$

$\boxed{F_1 = 9.97\,kN}$

(8) Determine the magnitude of F and angle θ, for a particle A shown in fig is subjected to forces gn to be in equilibrium.

Soln

Applying equilibrium conditions,



$\Sigma F_x = 0.$

$275\cos 15 - 225\cos 60 - 350\cos\theta = 0$

$350\cos\theta = 153.1296$

$\boxed{\theta = 64.05^\circ}$

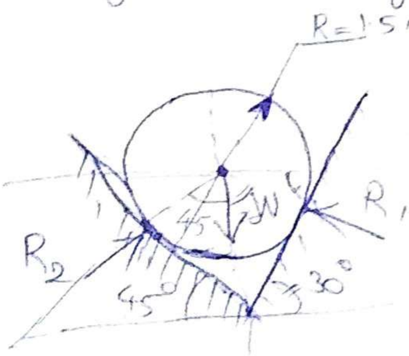
$\Sigma F_y = 0,$

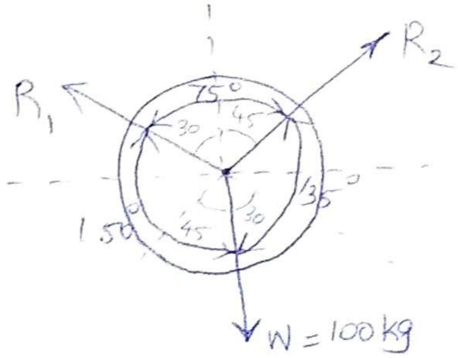
$F + 275\sin 15 + 225\sin 60 = 350\sin\theta$

$F = 350\sin 64.05 - 275\sin 15 - 225\sin 60$

$\boxed{F = 48.68\,N}$

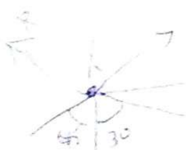
9 A smooth circular cylinder of radius 1.5 m is kept in a triangular groove, one side of which makes 45° and the other at 30° with horizontal. Find the reactions at the surface of contact, if there is no friction and the cylinder weight is 100 kg.





$= 100 \times 9.81\,N$

$W = 981\,N$



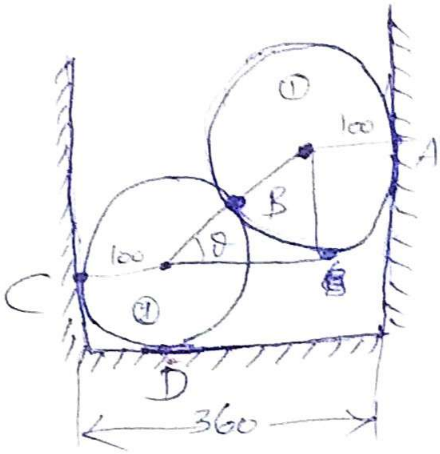
Applying Lami's theorem,

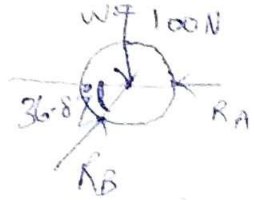
$$\frac{981}{\sin 75^\circ} = \frac{R_1}{\sin 135^\circ} = \frac{R_2}{\sin 150^\circ}$$

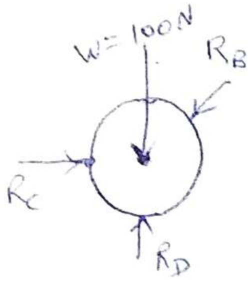
$$\boxed{R_1 = 718.14\ N \ ; \quad R_2 = 507.8029 N}$$

---

11) Two smooth sphere each of radius 100 mm and weight 100 N rest in a box having vertical sides, the distance between the sides being 360 mm. Find the reactions at the point of contacts A, B, C and D as shown in fig.







Let A, B, C & D are the points of contacts.

From schematic sketch,

$$\cos\theta = \frac{160}{200}$$

$$\theta = 36.87^\circ$$

Consider Sphere I :

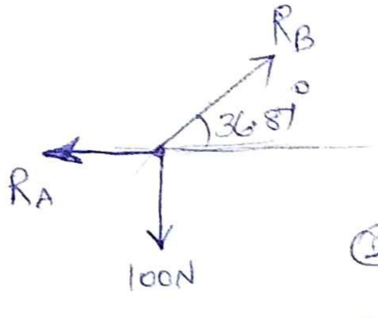
$\Sigma F_x = 0 \Rightarrow R_A = R_B \cos 36.87$

$\Sigma F_y = 0 \Rightarrow 100 = R_B \sin 36.87^\circ$

$$\boxed{R_B = 166.6662\ N}$$

$$\boxed{R_A = 133.3328\ N}$$

①



Consider sphere II :

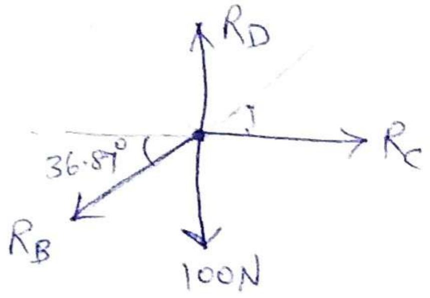
$\Sigma F_x = 0 \Rightarrow R_C = R_B \cos 36.87^\circ$

$$\boxed{R_C = 133.33 N}$$

$\Sigma F_y = 0 \Rightarrow R_D = R_B \sin 36.87^\circ + 100$

$$\boxed{R_D = 200 N}$$

②



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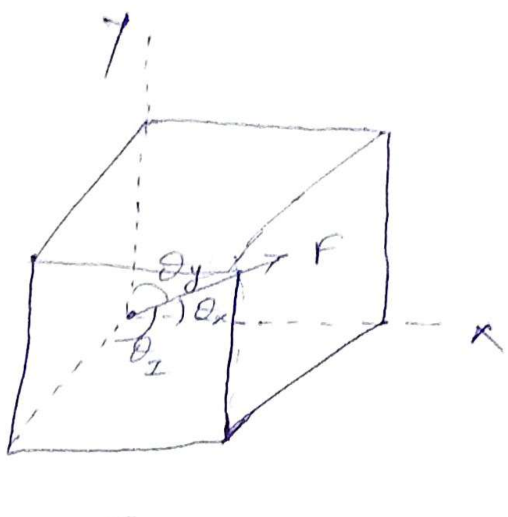
Resolution of Concurrent forces in Space (3D)

Consider a force F acting at the origin 'O' of the system of the rectangular co-ordinates, x, y & Z. The force F is making angles of θ_x, θ_y & θ_Z with x, y, Z axis.



The forces in the x direction $F_x = F\cos\theta_x$

" y " $F_y = F\cos\theta_y$

" Z " $F_Z = F\cos\theta_Z$

The force can be defined by the vector.

$$\boxed{F = F\cos\theta_x\, i + F\cos\theta_y\, j + F\cos\theta_Z\, k}$$

Magnitude of $F = \sqrt{F_x^2 + F_y^2 + F_Z^2}$

The cosine of θ_x, θ_y & θ_Z are known as the direction cosines of the force F and are defined by $l = \cos\theta_x$, $m = \cos\theta_y$; $n = \cos\theta_Z$

$$F = F(li + mj + nk) = F\hat{n}$$

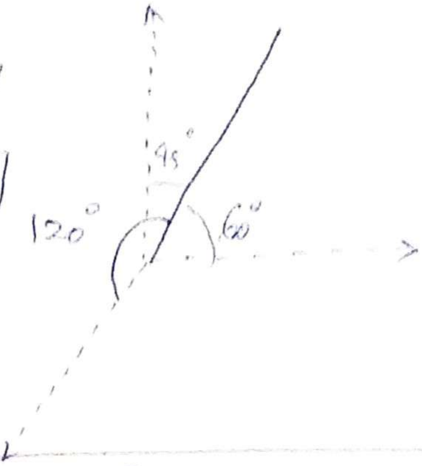
unit vector $\hat{n} = li + mj + nk$

$$l^2 + m^2 + n^2 = 1$$

and $\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_Z = 1$.

Numerical Problems

1. A force of 500 N forms an angles of $60°$, $45°$, $120°$ resp. with x, y & Z axes. Write the force in the vector form.



$$F = F\hat{n}$$

$$= F\left[\cos\theta_x i + \cos\theta_y j + \cos\theta_z k\right]$$

$$\bar{F} = 500\left[\cos 60 i + \cos 45 j + \cos 120 k\right]$$

$$\bar{F} = (250 i + 353.55 j - 250 k)\ N$$

2. A force F has the components $F_x = 20N$, $F_y = -30\ N$, $F_z = 60N$. Determine its magnitude F and angle θ_x, θ_y & θ_z it forms with the co-ordinate axes.

Force Vector $\bar{F} = F\hat{n}$

$$= F\left[\cos\theta_x i + \cos\theta_y j + \cos\theta_z k\right)$$

$$\bar{F} = F_x i + F_y j + F_z k$$

$$\bar{F} = 20 i + (-30) j + 60 k$$

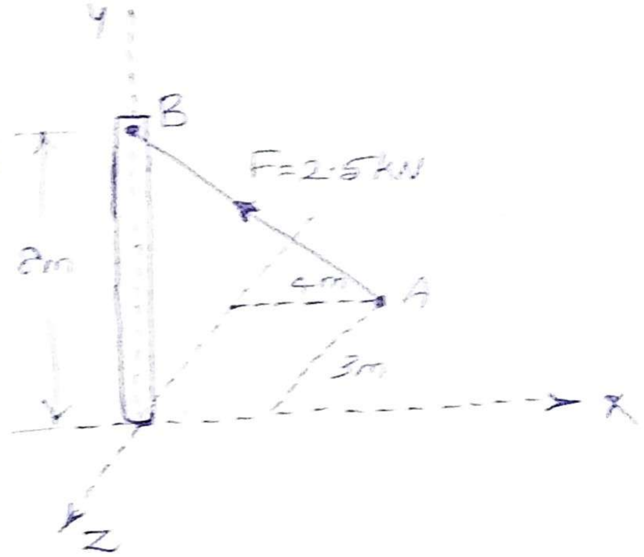
Magnitude of F $|F| = 70\ N$ $\left[\because \sqrt{20^2 + 30^2 + 60^2}\right]$

The angle $\theta_x = \cos^{-1}\left[\frac{20}{70}\right] = 73.398°$

$$\theta_y = \cos^{-1}\left[\frac{-30}{70}\right] = 115.376°$$

$$\theta_z = \cos^{-1}\left[\frac{60}{70}\right] = 31.002°$$

8. A guy wire AB is anchored by means of bolts at A. The force carried by the wire is 2.5 kN. Determine
a) The components of F_x, F_y and F_z of the force
b) The direction cosines and c) angle with all reference axes.



The co-ordinates of
A [4, 0, −3] and B [0, 8, 0]

$\overline{AB} = (0-4)i + (8-0)j + [0-(-3)]k$

$\overline{AB} = -4i + 8j + 3k$

$$\hat{n} = \frac{\overline{AB}}{|\overline{AB}|} = \frac{-4i+8j+3k}{\sqrt{4^2+8^2+3^2}}$$

$\hat{n} = -0.424i + 0.848j + 0.318k$
$= li + mj + nk$

$\overline{F} = F\hat{n}$

$\overline{F} = 2.5(-0.424i + 0.848j + 0.318k) = \cancel{F(li+mj+nk)}$

$\overline{F} = -1.06i + 2.12j + 0.795k$ (kN) $= F_x i + F_y j + F_z k$

$\boxed{F_x = -1.06\text{ kN};\ F_y = 2.12\text{ kN};\ F_z = 0.795\text{ kN}}$

Direction cosines $l = -0.424$
$m = 0.848$
$n = 0.318$

Angle with x axis, $\theta_x = \cos^{-1}(-0.424) = 115.087^\circ$
" y axis, $\theta_y = 32^\circ$
z axis, $\theta_z = 71.46^\circ$

Equilibrium of a Particle Subjected to Space Forces:-

When particle is in equilibrium, the resultant of the force system acting on it must be zero. $\bar{R} = 0$

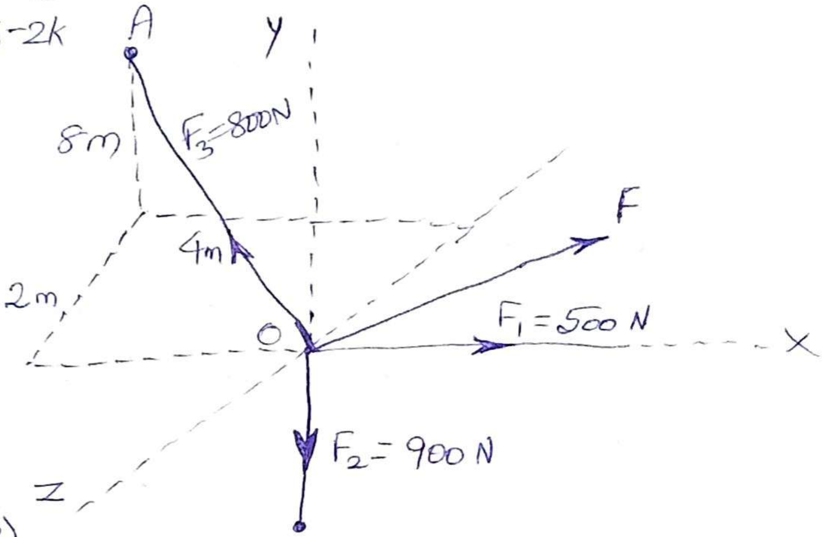
$$\Sigma F_x i + \Sigma F_y j + \Sigma F_z k = 0$$

$\therefore$ Equilibrium Conditions are, $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$

Numerical Problems:-

1. Determine the magnitude and the direction of force F shown in fig. A [-4 8 -2]

$A = -4i + 8j - 2k$



$F_1 = 500\, i\ N$

$F_2 = -900\, j\ N$

$$F_3 = 800 \frac{(-4i + 8j - 2k)}{\sqrt{16 + 64 + 4}}$$

$$F_3 = -349i + 698j - 174.57k$$

$F = F_x i + F_y j + F_z k$ (unknown force)

For equilibrium $\Sigma F = 0$ i.e, $R = 0$

Equating i, j and k components to zero,

$\Sigma F_x = 0 \Rightarrow 500 - 349 + F_x = 0$

$F_x = -151\ N$

$\Sigma F_y = 0 \Rightarrow -900 + 698 + F_y = 0$

$F_y = 202\ N$

$\Sigma F_z = 0 \Rightarrow -174.57 + F_z = 0$

$F_z = 174.54\ N$

$\bar{F} = -151i + 202j + 174.54k$ N

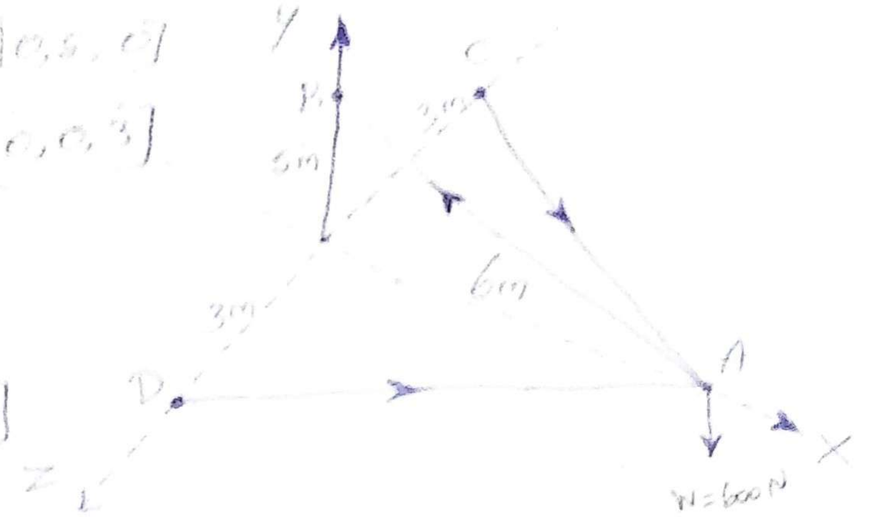
$F = 306.95$ N

$\theta_x = \cos^{-1}\left(\frac{-151}{306.95}\right) = 119.467^\circ$

$\theta_y = \cos^{-1}\left(\frac{202}{306.95}\right) = 48.84^\circ$

$\theta_z = \cos^{-1}\left(\frac{174.54}{306.95}\right) = 55.34^\circ$

2. A wall bracket consisting of bars AB, AC and AD as shown in fig are joined at the point A. Find the forces of the members meeting at A.



$A = [6,0,0]$; $B = [0,5,0]$

$C\ [0,0,-3]$; $D = [0,0,3]$

$$\overline{T_{AB}} = \frac{T_{AB}\,[-6i+5j]}{\sqrt{6^2+5^2}}$$

$$= T_{AB}\,[-0.77i+0.64j]$$

$$\overline{T_{CA}} = \frac{T_{CA}\,[6i+3k]}{\sqrt{6^2+3^2}} = T_{CA}\,[0.9i+0.45k]$$

$$\overline{T_{DA}} = \frac{T_{DA}\,[6i-3k]}{\sqrt{6^2+3^2}} = T_{DA}\,[0.9i-0.45k]$$

$$W = -600j\ (N)$$

Applying the equilibrium for particle the spaces.

$\Sigma F_x = 0 \quad \ldots \quad -0.77\,T_{AB} + 0.9\,T_{CA} + 0.9\,T_{DA} = 0$ — (1)

$\Sigma F_y = 0 \quad \ldots \quad \cancel{0.64j - 600j} \quad 0.64\,T_{AB} - 600 = 0.$

$T_{AB} = 937.5\ N$

$\Sigma F_z = 0 \quad \ldots \quad 0.45\,T_{CA} - 0.45\,T_{DA} = 0$

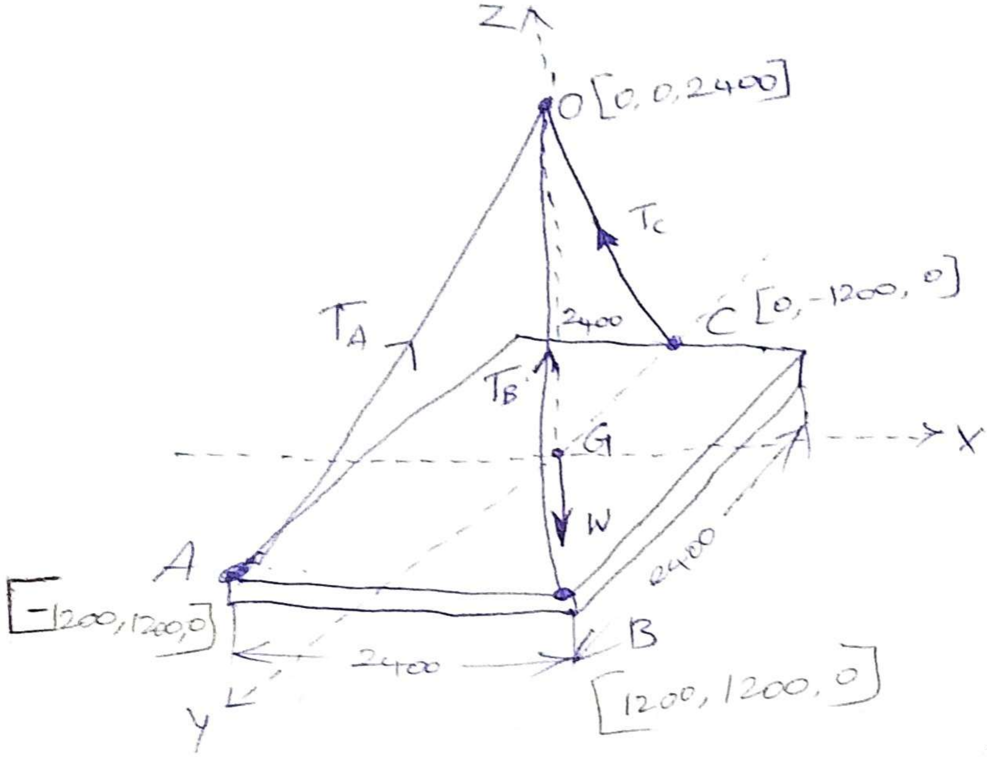
$T_{CA} = T_{DA}$ [Subs in (1)]

(1) $\Rightarrow \quad -0.77\,T_{AB} + T_{CA}\,(1.8) = 0$

Subs $T_{AB} = 937.5 \Rightarrow T_{CA} = 400.82\ N = T_{AD}$ //

6)

A Square plate has a mass of 1000kg with centre at 'G'. Calculate the tension in each cable when the plate lifted is horizontal?



$[-1.2i + 1.2j]$ $[0, 0, 2.4]$

~~OA =~~ ~~For~~

$OA = 1.2(-1.2j + 2.4k$

~~Position~~ vectors

$A = [-1.2i + 1.2j]$

$B = [1.2i + 1.2j]$

$C = [0, -1.2j]$

$O = [0, 0, 2.4]$

Tension $\overline{T_{AO}} = T_{AO} \dfrac{[1.2i - 1.2j + 2.4k]}{\sqrt{1.2^2 + 1.2^2 + 2.4^2}}$

$= T_{AO}[0.4082i - 0.4082j + 0.8165k]$

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$$\overline{T_{BO}} = T_{BO} \frac{(-1.2i - 1.2j + 2.4k)}{\sqrt{1.2^2 + 1.2^2 + 2.4^2}} = T_{BO}[-0.4082i - 0.4082j + 0.816$$

$$\overline{T_{CO}} = T_{CO} \frac{1.2j + 2.4k}{\sqrt{1.2^2 + 2.4^2}} = T_{CO}[0.447j + 0.8944k]$$

$W = -9.81k$ (kN)

$W = -9810k$ (N)

1000 kg × 9.81 N
9810 N
9.81 kN

When the plate is in equilibrium,

$\Sigma F_x = 0 \quad 0.4082\, T_{AO} - 0.4082\, T_{BO} = 0$

$T_{AO} = T_{BO} = T$ ——— ①

$\Sigma F_y = 0 \quad \cancel{-0.4082j - 0.4082}$

$-0.4082\, T_{AO} - 0.4082\, T_{BO} + 0.447\, T_{CO} = 0$

$T[-0.8164] = -0.447\, T_{CO}$

$T_{CO} = \frac{0.8164\, T}{0.447}$

$T_{CO} = 1.8264\, T$ ——— ②

$\Sigma F_z = 0 \Rightarrow 0.8165\, T_{AO} + 0.8165\, T_{BO} + 0.8944\, T_{CO} - 9810 = 0$

Apply ① & ② $\quad 1.633\,T + 0.8944 \times 1.8264\,T = 9810$

$3.26652\,T = 9810$

$\boxed{T = 3003.18\ N} \Rightarrow \boxed{T_{AO} = T_{BO} = 3003.18\ N}$

② $\Rightarrow \boxed{T_{CO} = 5485.0168\ N}$

Moment

The force acting on a rigid body has not only the tendency to translate but it has a tendency to rotate the body.

The turning effect of a force measured by a quantity called Moment.

$\therefore \; M = F \times d$

The unit of moment is Nm.

Example:

Push or pull of a door by holding the handle about the hinge. Applying force to a spanner, when turning a nut while tightening.

The effect depends on the magnitude of the force and the $\perp r$ distance called moment arm.

Free Body Diagram

It is a diagram which shows all the forces acting at a rigid body involving,

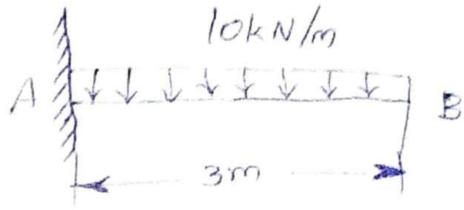
1. Self weight
2. Normal reactions
3. Frictional force
4. Applied force
5. External moment applied.

Types of Loads:

1. Concentrated or Point load

If a load is acting at a particular point, then the load is called point load or concentrated load.

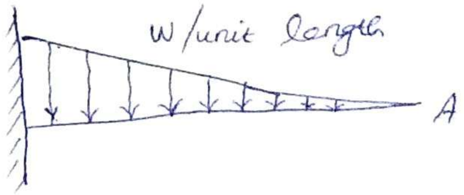
2. Uniformly distributed Load (UDL)



If the load is distributed uniformly through a length, it is called 'Uniformly distributed load'.

Example: Self weight of the beam or shaft. Here a UDL of 10 kN/m is acting throughout a distance of 3 m length. Therefore the total load is 3×10 = 30 kN which is acting at a centroid distance of 1.5 m from fixed end.

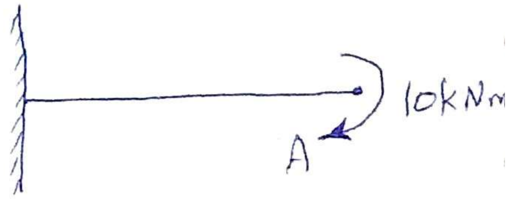
3. Uniformly Varying Load



The intensity of the load is varying from a point to another point linearly as shown.

Here the load intensity is w/unit length at fixed end and it decreases uniformly to zero at 'A'.

4. External Moment:



The applied moment is also a type of load. The fig shows a moment of 10 kNm acting on a beam at 'A'.

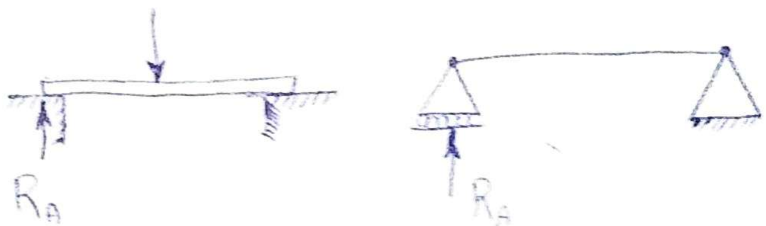
Types of Supports and Reactions

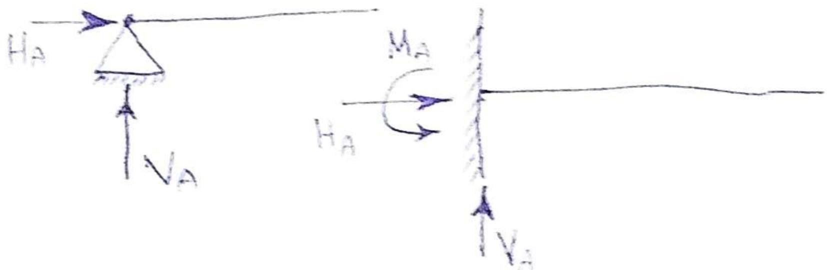
Constraints: When the free movement in a particular direction is restricted, the support is called Constraints, Where the reactions is ~~applie~~ developed.

A point in space has 6 degrees of freedom. They are translation in x, y & z directions and rotation about x, y & z axes. Therefore, if any one degree of freedom is restricted, a reaction will be introduced while drawing the free body diagram.

Types of Supports in two dimensions

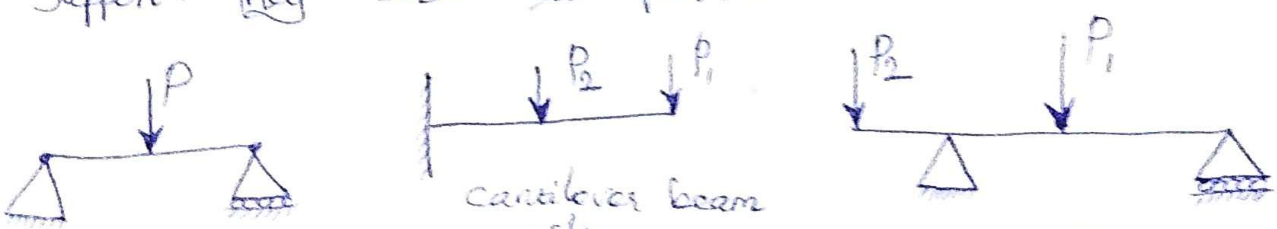
1. Simple Support
2. Roller Support
3. Hinged Support
4. Fixed Support





Beam:

Beam is a horizontal structural member subjected to transverse load. It gets deflection when the loads are applied. Beams are classified on the basis of support. They are as follows

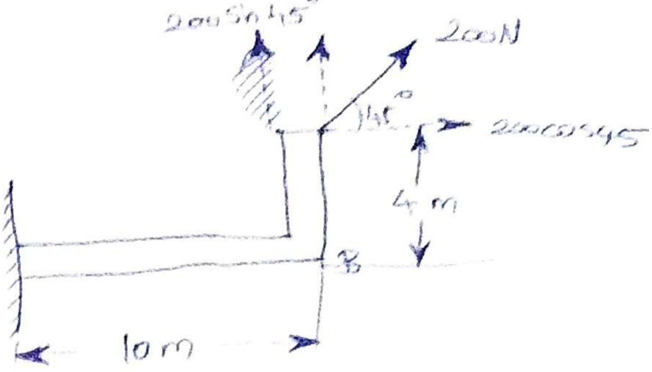


(a) SSB

cantilever beam (b)

(c) Overhanging beam

1. Determine the moment of a force about A & B.



Taking moment about 'B'

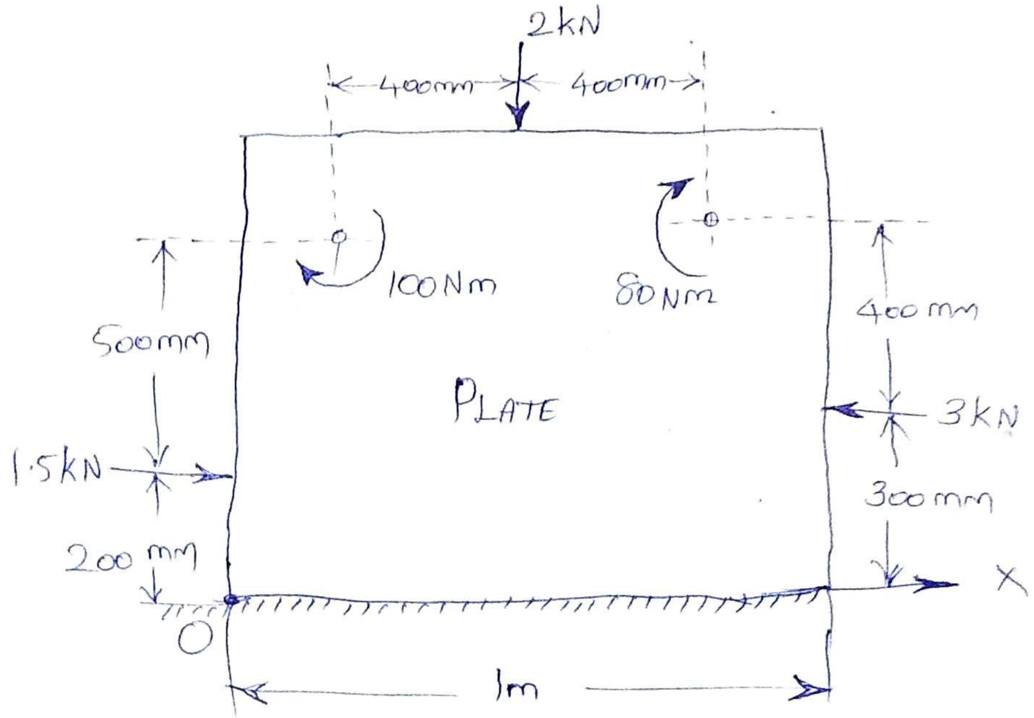
$-200 \cos 45° \times 4 = -565.685$ Nm

$M_{B} = 565.685$ N·m (cw)

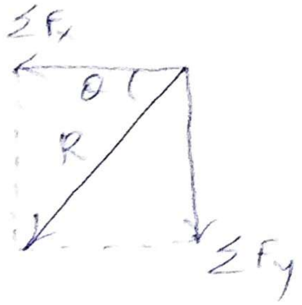
Taking Moment about 'A'

$-200 \cos 45 \times 4 + 200 \sin 45 \times 10 = 848.528$ (ccw)

2. The plate is acted upon by three forces and two couples as shown in fig. Determine the resultant of these force couple system and find co-ordinate x ~~and~~ of the point on the x axis through which the resultant is passed.



$\Sigma F_x = 1.5 - 3 = -1.5\ kN$



$\Sigma F_y = -2\ kN$

$R = \sqrt{1.5^2 + 2^2} = 2.5\ kN$

$\theta = \tan^{-1}\left(\frac{2}{1.5}\right) = 53.13^\circ$

Taking moment about O,

$$M_o = -2(0.5) + 3(0.3) - 1.5(0.2) - 0.1 - 0.08$$

$$= -0.58\ kNm$$

$d = \frac{\Sigma M}{R} = \frac{0.58}{2.5}$

$\boxed{d = 0.232\ m}$

$M_o = 0.58\ (CW)$

The Co-ordinate x of the point on x axis through which the resultant passes is gn by,

$$x = \frac{M_o}{\Sigma F_y} = \frac{+0.58}{+2} = 0.29\ m$$

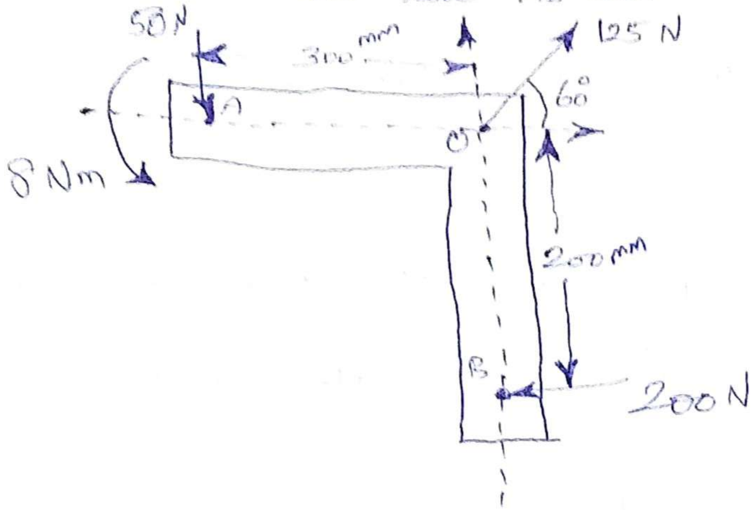
$\boxed{x = 290\ mm}$

If we want to find the Intersection,

$$y = \frac{M_o}{\Sigma F_x} = \frac{+0.58}{1.5} = 0.387\ m$$

$\boxed{y = 387\ mm}$

The three forces and a couple shown in fig. are applied to an angle bracket.
i) Find the resultant of this system of forces
ii) Locate the points where the line of action of the resultant intersect the line AB and line BC.



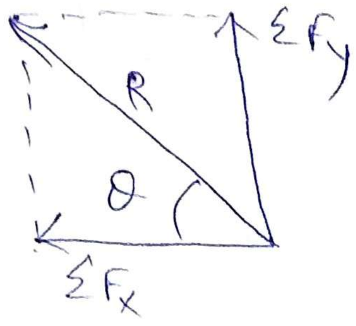
$\Sigma F_x = -200 + 125\cos 60 = -137.5\ N$

$\Sigma F_y = 125 \sin 60 - 50 = 58.2532\ N$

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = 149.33\ N.$

$\theta = \tan^{-1}\left(\frac{58.2532}{137.5}\right)$

$\boxed{\theta = 22.96^\circ}$



Moment about 'O' $M_o = (50 \times .300) - (200 \times 0.2) + 8$

$M_o = -17\ Nm$

$M_o = 17\ Nm\ (cw)$

The force couple system at O will be reduced into a single force at a point located at the ⊥r dist

$$d = \frac{\Sigma M}{R} = \frac{17}{149.33} = 0.11384 \text{ m}$$

$$d = 113.84 \text{ mm}$$

~~x & y be intersection of line of action of single force on the line AB & BC~~

The intersection points also be found by or checked

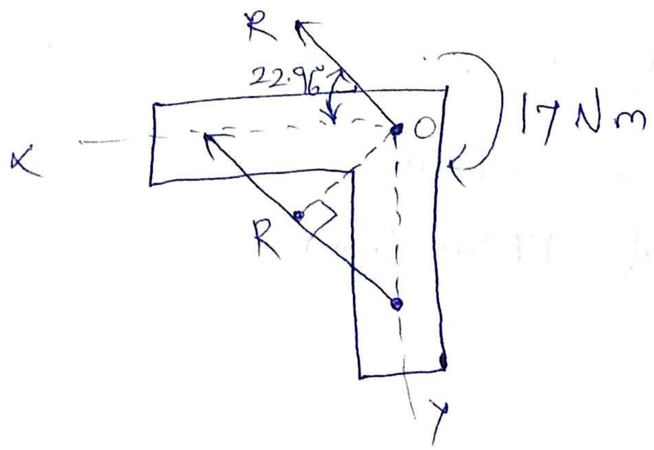
$$x = \frac{\Sigma M_o}{\Sigma F_y} = \frac{17}{58.253} = 291.83 \text{ mm}$$

$$y = \frac{\Sigma M_o}{\Sigma F_x} = \frac{17}{137.5} = 123.6 \text{ mm}$$

(or)

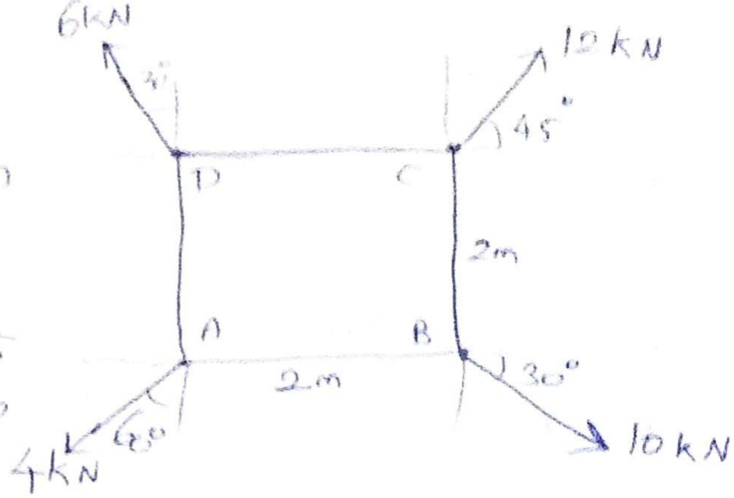
$$x = \frac{d}{\sin\theta} = \frac{113.84}{\sin 22.96} = 291.83 \text{ mm}$$

$$y = \frac{d}{\cos\theta} = \frac{113.84}{\cos 22.96} = 123.63 \text{ mm}$$



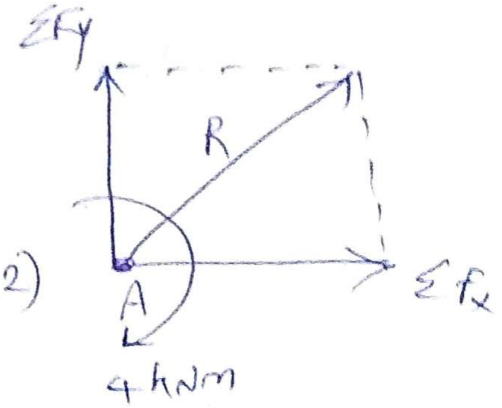
8. A Square plate of 2 m side is acted upon by forces as shown in fig. Find the resultant in magnitude, direction and also point of application with respect to the corner A.

$\Sigma F_x = 6\cos 60 + 12\cos 45$
$+ 10\cos 30 - 4\cos 30$
$= 10.68 kN$



$\Sigma F_y = 6\sin 60 + 12\sin 45$
$- 4\sin 30 - 10\sin 30$
$= 6.681 kN$

$R = 12.597$ kN
$\theta = 32.02°$



$\Sigma M_A = (6\cos 60 \times 2) - (12\cos 45 \times 2)$
$- (10\sin 30 \times 2) + (12\sin 45 \times 2)$
$= -4$ kNm

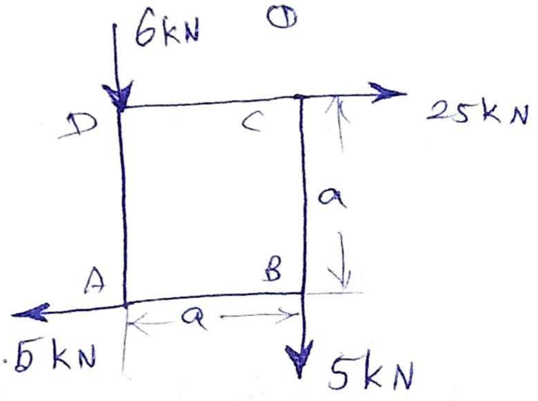
$\Sigma M_A = 4$ kNm (CW)

$d = \frac{\Sigma M_A}{R} = \cancel{0.5987 m}$ 0.3175 m

$x = \frac{\Sigma M_A}{\Sigma F_y} = 0.5987 m$

$y = \frac{\Sigma M_A}{\Sigma F_x} = 0.3745 m$

9 Determine the magnitude and direction of a single force P, which keeps the system in equilibri[um]. The system of forces acting is shown in fig. 1.



Soln:-

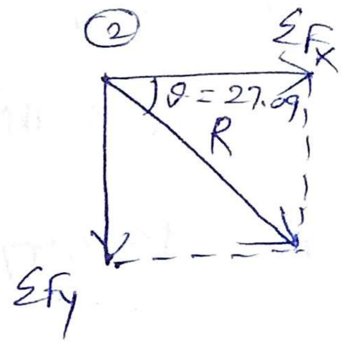
To find, 'E'.

$\Sigma F_x = 25 - 3.5 = 21.5\, kN$

$\Sigma F_y = -6 - 5 = -11\, kN.$

$R = 30.55\, kN$

$\theta = \tan^{-1}\left(\frac{11}{21.5}\right) = 27.0955^\circ$



$M_A = (\cancel{-}5 \times a) - (25 \times a) = -30a$

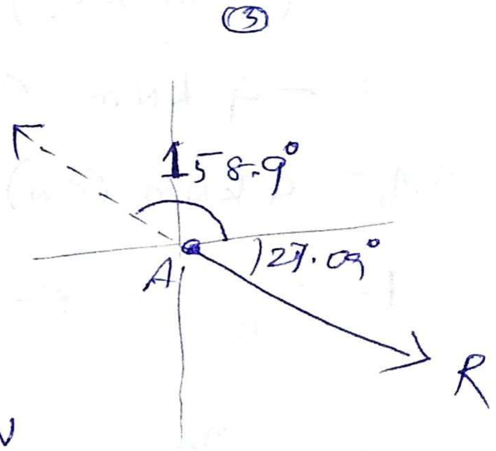
$M_A = 30a\,(CW)$

$d_{AE} = \frac{\Sigma M_A}{R}$

$x = \frac{\Sigma M_A}{\Sigma F_y}$

$y = \frac{\Sigma M_A}{\Sigma F_x} =$

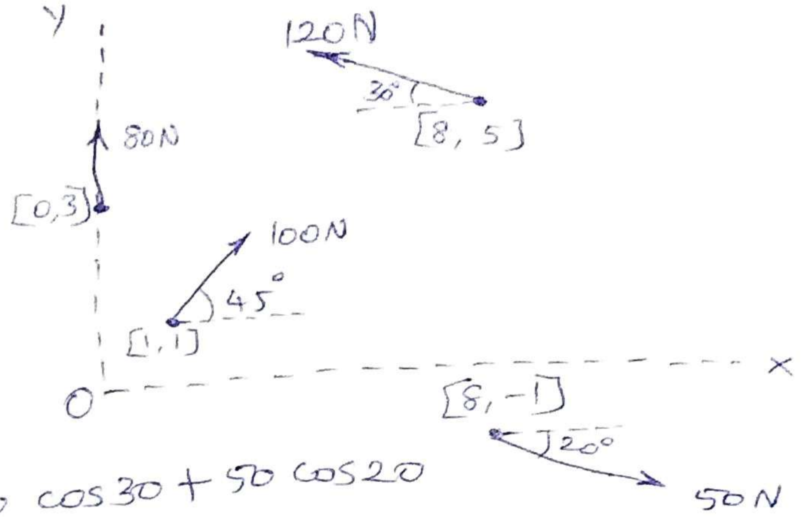
$= \frac{30a}{30.55}$



$d_{AE} = 0.981996\, a$

The equilibrant $E = 30.55\, kN$ acting at E with angle 158.9° with the x axis as shown in fig. 3

14. Determine the resultant of the non concurrent, non parallel system of forces shown in fig.

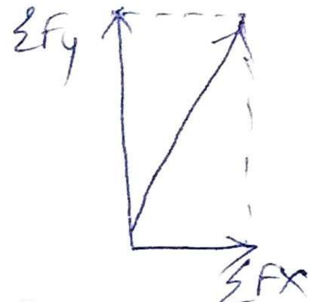


$\Sigma F_x = 100\cos 45 - 120\cos 30 + 50\cos 20$

$= 13.77$ N

$\Sigma F_y = 80 + 100\sin 45 + 120\sin 30 - 50\sin 20$

$= 193.60967$ N



$R = 194.0987$ N

$M_o = (-100\cos 45 \times 1) + (100\sin 45 \times 1)$

$(120\cos 30 \times 5) + (120\sin 30 \times 8)$

$(50\cos 20 \times 1) - (50\sin 20 \times 8)$

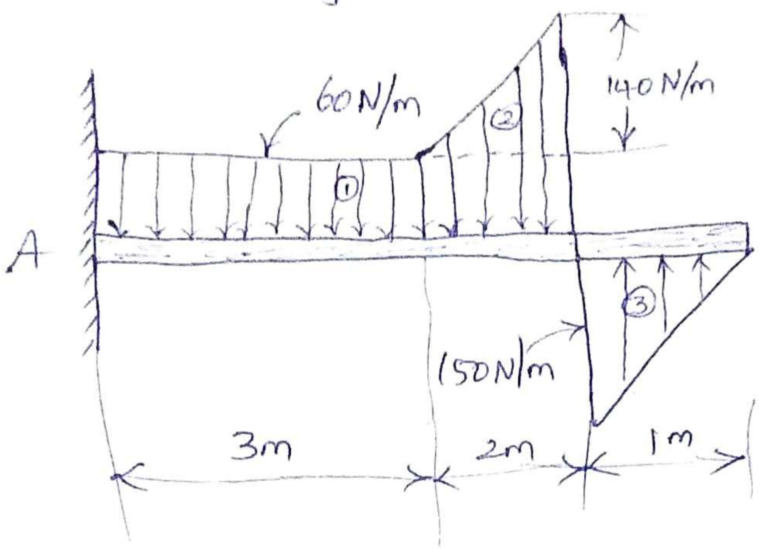
$\Sigma M_o = 909.79$ Nm (ccw)

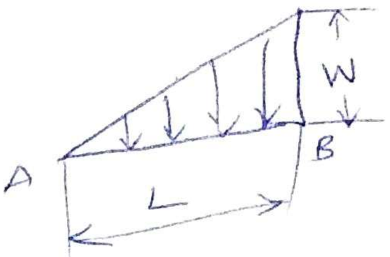
Now the force and couple at 'O' is deduced into a single resultant at (x, y). The ordinate of resultant is given by $d = \frac{\Sigma M_o}{R}$

$x = \frac{\Sigma M_o}{\Sigma F_y} = \frac{909.79}{193.60967} = 4.699$ m

$y = \frac{\Sigma M_o}{\Sigma F_x} = \frac{909.79}{13.77} = 66.07$ m

15. Find the single resultant force for the system of loading acting on the beam as shown in fig.



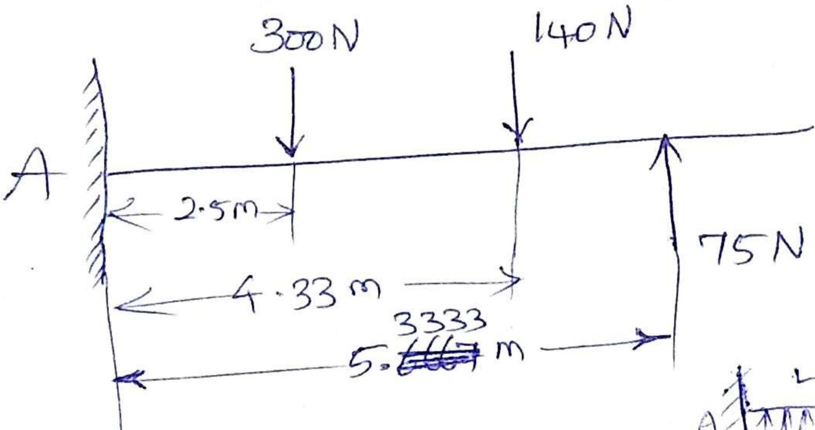


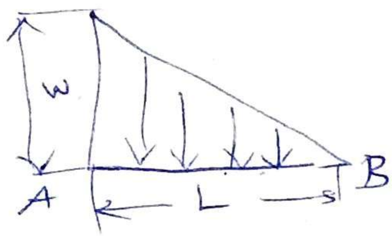
$\frac{1}{2}(LW)$ is acting $\frac{2}{3}L$ distance from 'A'

Load 1 having magnitude of $\left[(60\times5)\times\cancel{1}\right]$ N (↓)
acting at $\frac{5}{2}$ m from 'A'.

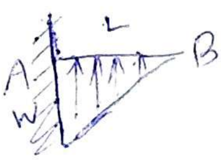
Load 2 having magnitude of $\left(\frac{1}{2}\times2\times140\right)$ N (↓)
acting at $3+\left(\cancel{\frac{2\;100}{3}}\right)$ m from 'A'
$\frac{2}{3}(2)$

Load 3 having magnitude of $\frac{1}{2}(1)(150)$ N (↑)
acting at $3+2+\left(\frac{1}{3}\right)$ m from 'A'.
$\left[1-\frac{2}{3}(1)\right]$





$\frac{1}{2}(L)(W)$ is acting $\left(L-\frac{2}{3}L\right)$ distance from A



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$R = F_y = -300 - 140 + 75 = -365\,N = 365 \downarrow$

$\Sigma M_A = -300 \times 2.5 - 140 \times 4.33 + 75 \times 5.33$

$= -956.45\, N.m$

$= 95645\, Nm$ (CW)

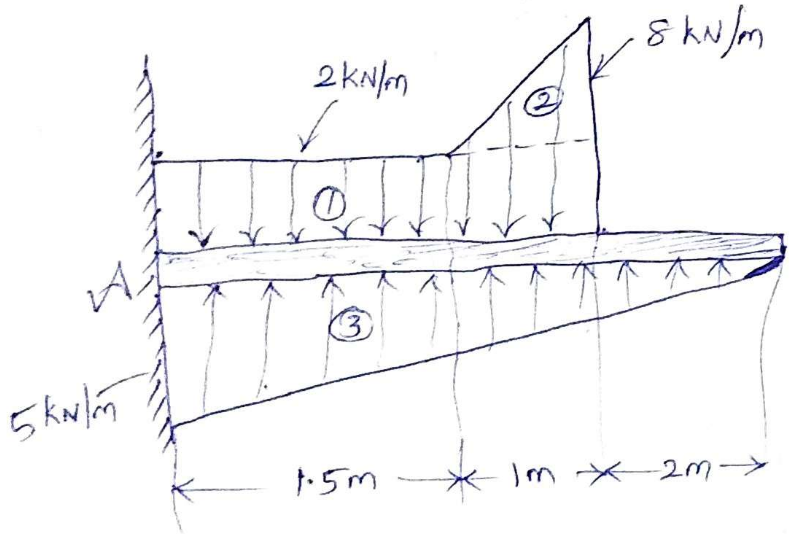
Position of single force R at a distance $d = \frac{\Sigma M_A}{R}$

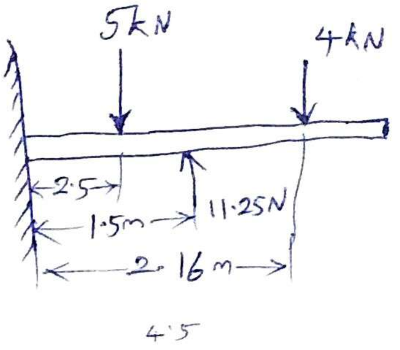
$d = \frac{956.45}{365}$

$d = 2.62\, m$ from $A(\downarrow)$

---

16. Find the simplest equivalent force for the system of loading acting on the beam shown in fig.





Load ① is having magnitude of $(2 \times 2.5)\,N\,(\downarrow)$ acting at $\frac{2.5}{2}$ from 'A'

Load ② is having magnitude of $\frac{1}{2}(1)(8)\,N\,(\downarrow)$ acting at $1.5 + (1 - \frac{1}{3})\,m$ from 'A'.

Load ③ is having magnitude of $\frac{1}{2}(4.5)(5)\,N\,(\uparrow)$ acting at $\frac{4.5}{3}m$ from 'A'.

Sum of forces $\Sigma R = \ldots 11.86 = 2.25\ kN\ (\uparrow)$ is a single resultant

Moment about A $\Sigma M_A = 11.00 \times 1.5 - 4 \times 1.25 - \frac{4}{5} \times 2.167$

$= 1.947\ kN.m$

The distance $d = \frac{M_A}{\Sigma R} = \frac{1.947}{2.24} = 0.86978\ m$ from 'A'.

Equilibrium of Rigid body Subjected to Non concurrent Coplanar force system.

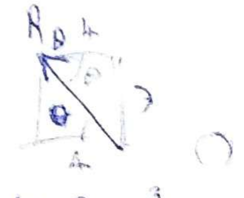
$\Sigma F_x = 0$ — Force towards right side = Force towards left side

$\Sigma F_y = 0$ — Upward force = Downward force

$\Sigma M = 0$ — Clockwise moment = Anticlockwise moment

∴ The equilibrium conditions of a rigid body are as follows

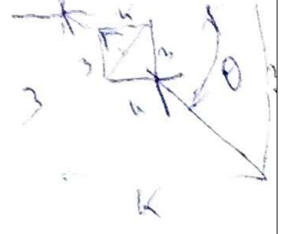
1. Sum of Horizontal forces is zero.
2. Sum of Vertical forces is zero.
3. Moment about any point is zero.

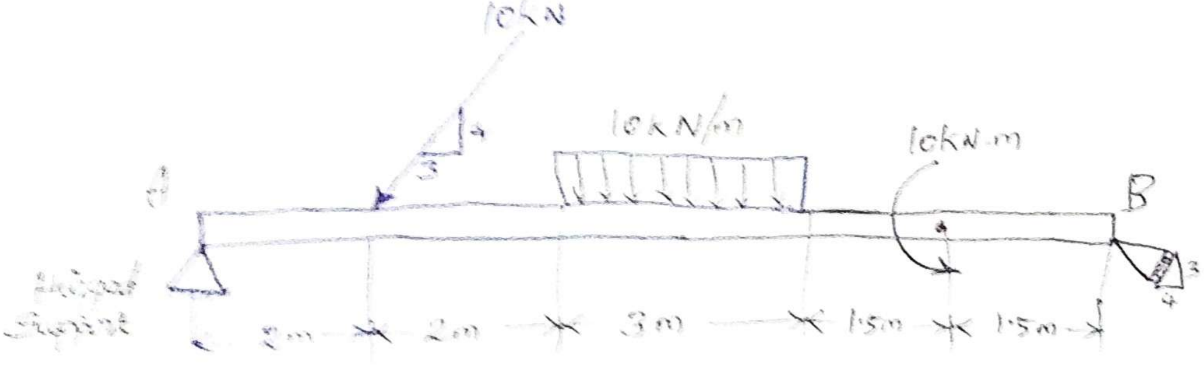


$\tan\theta = \frac{3}{4}$
$\theta = 36.57°$

Numerical problems:

1. Find the reaction at the supports A & B for a beam as shown in fig.





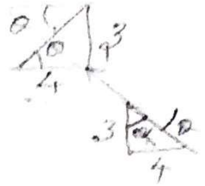
$R_B \sin 36.87°$

$R_B \cos 36.87$

$\tan\theta = \frac{4}{3}$
$\theta = \tan^{-1}\frac{4}{3} = 53.13°$

At and B, The reactions,

~~$\tan\theta = \frac{4}{3}$~~



$$\tan\theta = \frac{4}{3}$$

$$\theta = 53.13^\circ$$

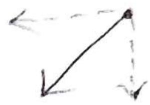
Horizontal force $= 10\cos\theta$
$= 6\ kN$

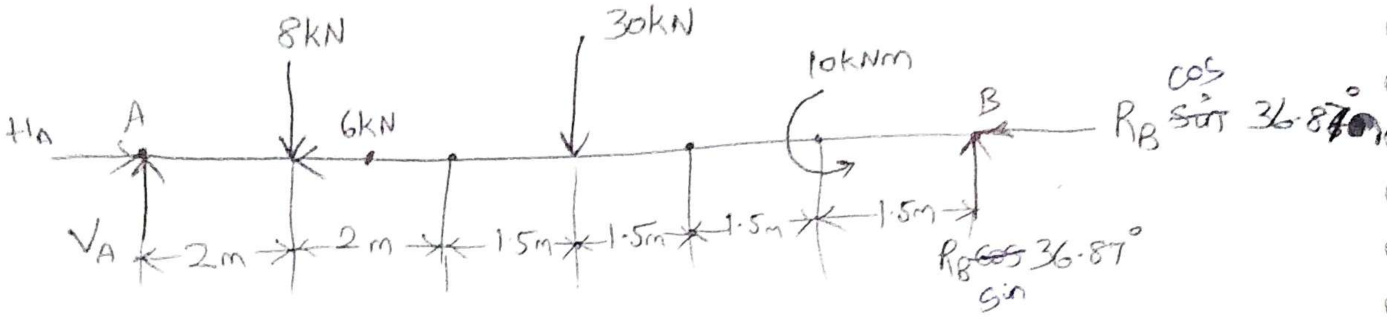
Vertical force $= 10\sin\theta$
$= 8\ kN$

$$\tan\alpha = \frac{4}{3} \Rightarrow \alpha = 53.13^\circ$$

$$\theta = 90 - \alpha = 36.86989^\circ$$

Free body diagram:-





Applying equilibrium conditions,

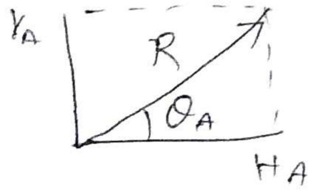
$$\Sigma F_x = 0 \Rightarrow H_A - 6 - R_B \sin 36.87^\circ = 0 \quad \text{---} (1)$$

$$\Sigma F_y = 0 \Rightarrow V_A - 8 - 30 + R_B \cos 36.87^\circ = 0 \quad \text{---} (2)$$

$$M_A = 0 \Rightarrow -(8\times2) - (30\times5.5) + 10 + (R_B \cos 36.87^\circ \times 10) = 0 \quad \text{---} (3)$$

$$\boxed{R_B = 21.375\ kN}$$

Subs R_B value in (1) & (2)



(1) $\Rightarrow$ $H_A = 18.825\ kN$

eqn (2) $\Rightarrow$ $V_A = 20.9\ kN$

$$R_A = \sqrt{H_A^2 + V_A^2} = 28.128^\circ$$

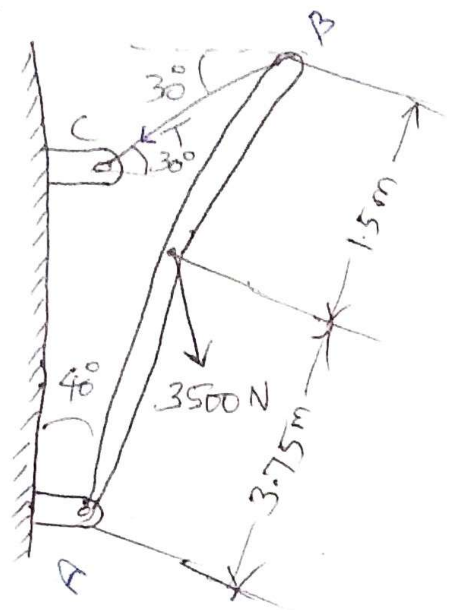
$$\boxed{\begin{aligned}\theta_A &= \tan^{-1}\left(\frac{20.9}{18.825}\right) \\ \theta_A &= 47.99^\circ \text{ with Horizontal}\end{aligned}}$$

4. A load P of 3500 N is acting on the boom, which is held by cabled BC as shown in fig. The weight of the boom can be neglected.

a) Draw the free body diagrams of the boom

b) Find the tension in cable BC

c) Determine the reaction of A.



△ ADE,

1. $\cos\theta = \dfrac{\text{Adj. side (AE)}}{3.75}$

$\cos 50 = \dfrac{\text{Adj. Side (AE)}}{3.75}$

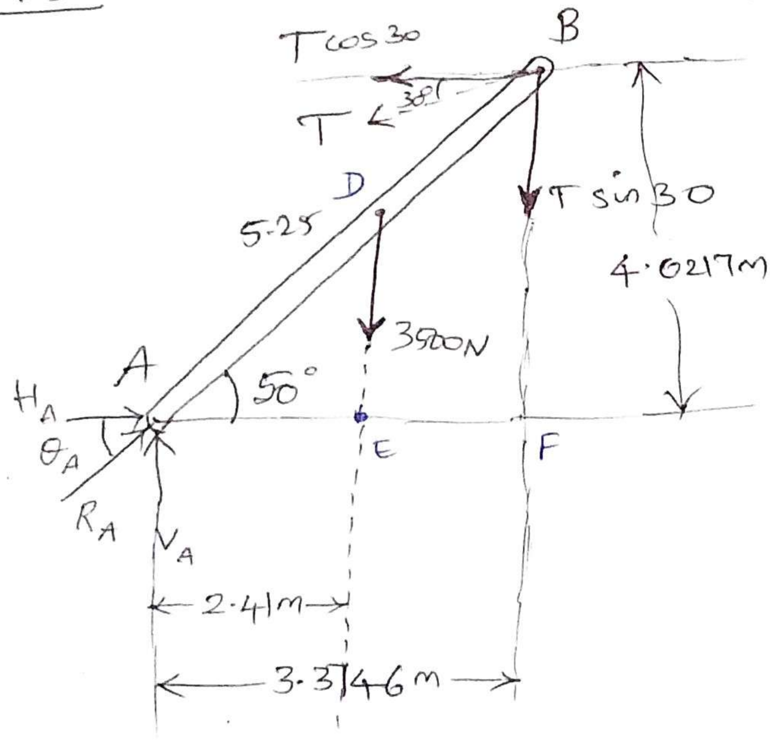
Adj. side (AE) = 2.41 m

△ ABF,

2. $\cos 50 = \dfrac{\text{Adj. side (AF)}}{5.25}$

Adj. side (AF) = 3.3746 m

FBD



△ ABF,

3. $\sin 50 = \dfrac{\text{Opp. side (BF)}}{5.25}$

Opp. side (BF) = 4.0217 m

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$$\Sigma M_A = 0$$

$$\Sigma M_A = -(3500 \times 2.41) - (T\sin 30 \times 3.3746) + (T\cos 30 \times 4.0217)$$

$$-8435 - 1.6873T + 3.4829T = 0$$

$$1.7956T = 8435$$

$$\boxed{T = 4697.6\,N}$$

$$\Sigma F_x = 0$$

$$H_A - T\cos 30 = 0$$

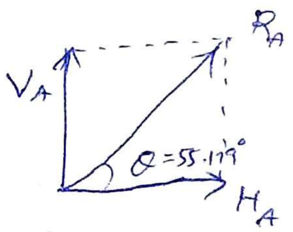
$$T\cos 30 = H_A$$

$$H_A = 4068.2486\,N$$

$$\Sigma F_y = 0$$

$$V_A = T\sin 30 + 3500$$

$$V_A = 5848.8\,N$$



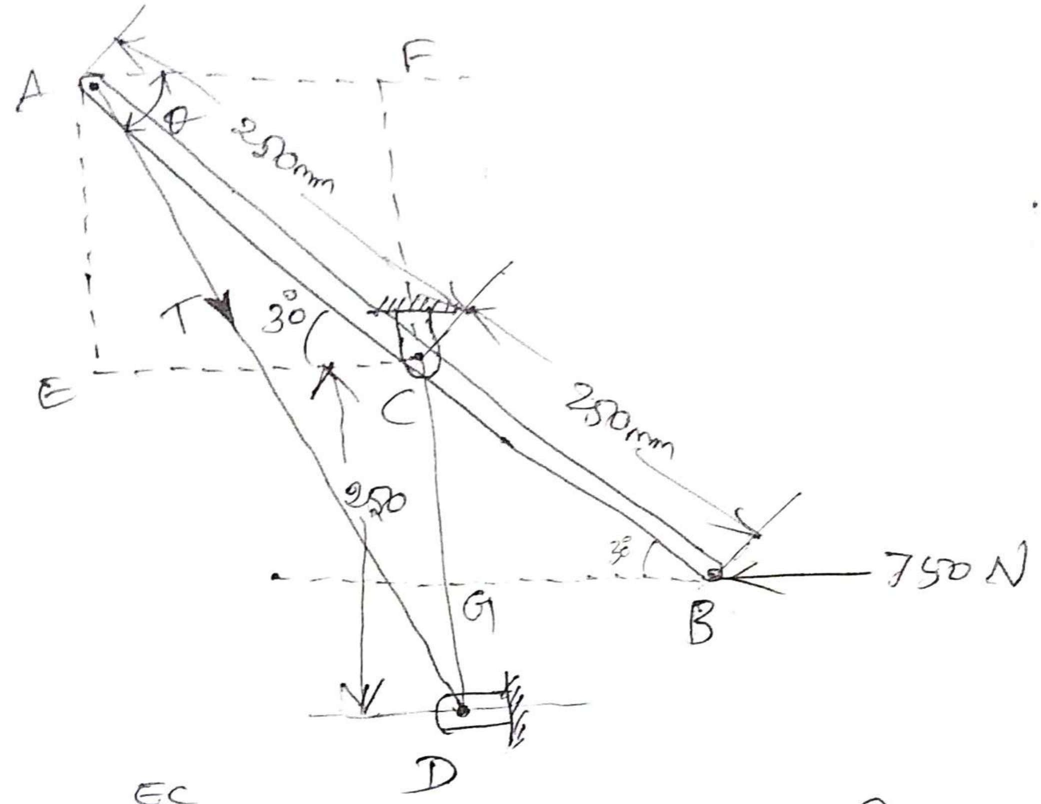
$$R_A = 7124.54\,N$$

$$\theta_A = \tan^{-1}\left(\frac{5848.8}{4068.2486}\right)$$

$$\boxed{\theta_A = 55.179°}$$

7. In the fig. lever AB is hinged at C and attached to a cable at A. If the lever is subjected to a horizontal force of 750 N at B, determine,

a) tension in the cable b) reaction at hinged supp



$\triangle ACE$,

(i) $\cos 30 = \frac{EC}{250} \Rightarrow EC = 216.5 \text{ mm} = AF$ ✓

(ii) $\sin 30 = \frac{AE}{250} \Rightarrow AE = 125 \text{ mm} = CF$ ✓

(iv) $\triangle ADF$,

$$\tan\theta = \frac{CF + CD}{AF}$$

$$\tan\theta = \frac{125 + 250}{216.5}$$

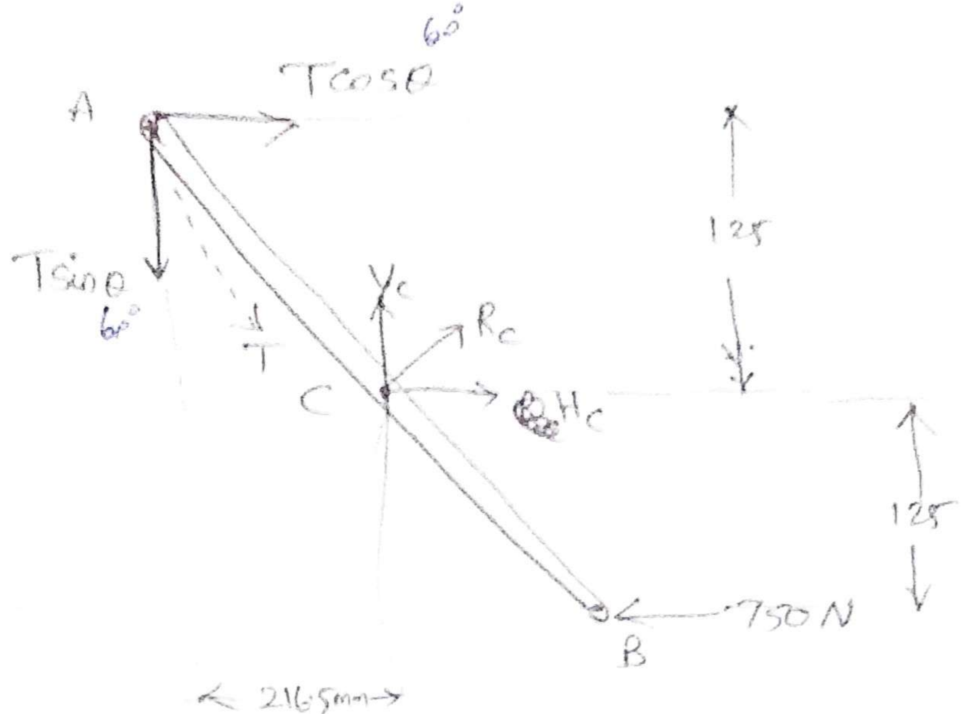
$$\theta = \tan^{-1}(1.7321)$$

$$\boxed{\theta = 60^\circ}$$

(iii) $\triangle CGB$

$$\sin 30 = \frac{CG}{250}$$

$$CG = 125 \text{ mm}$$



Applying equilibrium conditions,

$$\Sigma M_A = 0 \Rightarrow (V_c \times 216.5) + (H_c \times 125) - (750 \times 250) = 0$$

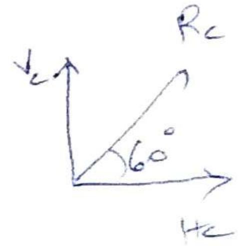
$$\text{(i) } \Sigma M_c = 0 \Rightarrow -(T\cos 60 \times 125) + (T\sin 60 \times 216.5)$$

$$-(750 \times 125) = 0$$

$$-62.5T + 187.49T = 93750$$

$$\cancel{T = 375N}$$

$$\boxed{T = 750\text{ N}}$$ Ans



$$\Sigma F_x = 0 \Rightarrow T\cos 60 + H_c - 750 = 0$$

$$\boxed{H_c = 375N}$$

$$\Sigma F_y = 0 \Rightarrow -T\sin 60 + V_c = 0 \Rightarrow \boxed{V_c = 649.52\text{ N}}$$

$$\boxed{R_c = 750N}$$

$$\boxed{\theta_c = 60^\circ}$$ Ans

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = 60^\circ$$

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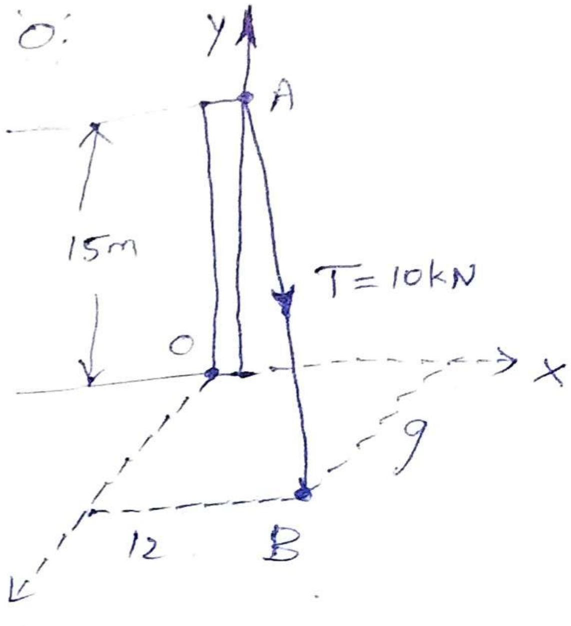
Resultant of Non-concurrent Space Force System:
(3D Rigid Body)

1. A tension T of magnitude 10 kN is applied to the cable attached to the top A of rigid mass and secured to the ground at B as shown in fig. Determine moment of tension T about Z axis passing through the base O.

Soln:



The position vector of

A=[0, 15, 0], B=[12, 0, 9]

Expressing the T as vector.

$\overline{AB} = 12i - 15j + 9k$

$\hat{n} = \frac{\overline{AB}}{|AB|} = \frac{12i - 15j + 9k}{\sqrt{12^2 + 15^2 + 9^2}}$

$\hat{n} = 0.565685i - 0.7071j + 0.42426k$

$\overline{T} = T\,\hat{n}_T$

$= 10[\hat{n}_T]$

$\overline{T} = 5.65685i - 7.071j + 4.2426k$

Moment of T about O, $\overline{M}_o = \overline{r}_{OA} \times \overline{T}$

$= \begin{vmatrix} i & j & k \\ 0 & 15 & 0 \\ 5.65685 & -7.07 & 4.2426 \end{vmatrix}$

$= i[(15 \times 4.2426) - 0] - j[0 - 0] + k[0 - (15 \times 5.65685)]$

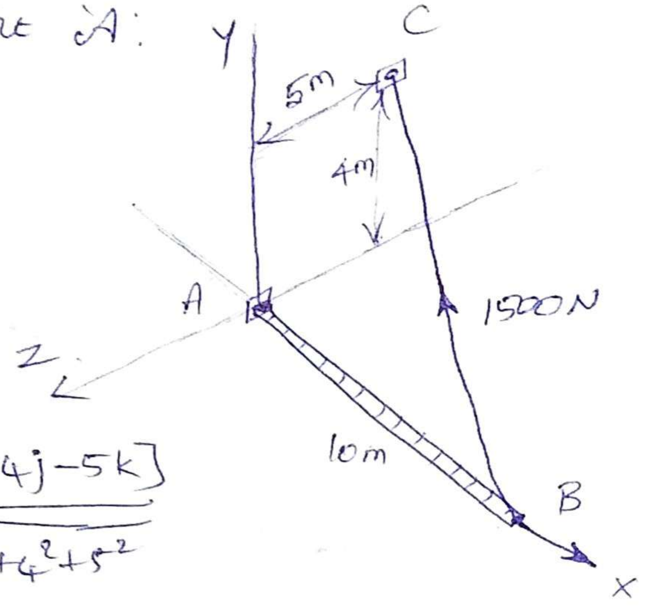
$\overline{M}_o = 63.639i - 84.85275k$ (kNm)

2. A 10m rod AB is shown in fig. A is a fixed end. A Steel cable is stretched from B to point 'C' on vertical wall. If the tension in the cable is 1500N. Find the moment of force exerted by the cable at 'B' about the point 'A'.

$A = [0, 0, 0]$

$B = [10, 0, 0]$

$C = [0, 4, -5]$



$$\overline{T_{BC}} = T_{BC} \frac{[-10i + 4j - 5k]}{\sqrt{10^2 + 4^2 + 5^2}}$$

$$= 1500 [\hat{n}]$$

$$\overline{T_{BC}} = -1263i + 505.5j - 631.61k \text{ (N)}$$

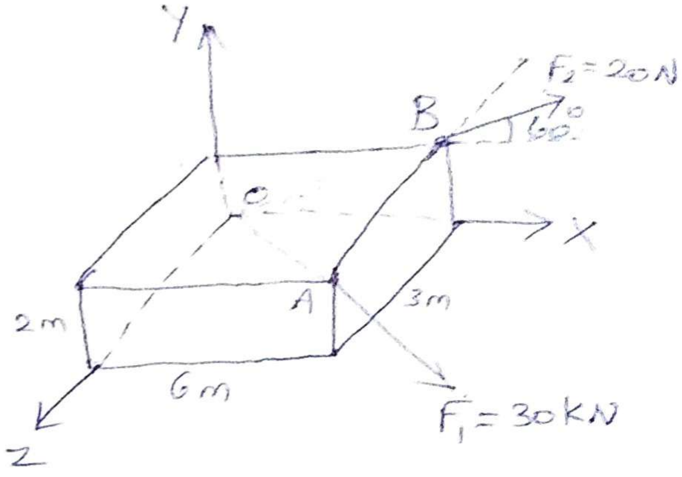
Moment of force exerted by the tension about A

$$\overline{M_A} = \overline{r_{AB}} \times T_{BC}$$

$$= \begin{vmatrix} i & j & k \\ 10 & 0 & 0 \\ -1263 & 505.5 & -631.61 \end{vmatrix}$$

$$\overline{M_A} = 6316.1j + 5055k \text{ (Nm)}$$

Two forces are acting on a slab as shown in fig. Replace the force system into an equivalent force - couple system at O.



Soln:-

The position vectors of A & B

O [0, 0, 0] A [6, 2, 3]

$\bar{r}_{OA} = 6i + 2j + 3k$

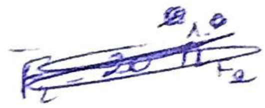
O [0, 0, 0] B [6, 2, 0]

$\bar{r}_{OB} = 6i + 2j$

$\bar{F}_1 = F_1 \hat{n}_{F_1}$

$$\bar{F}_1 = 30 \frac{[6i + 2j + 3k]}{\sqrt{36+4+9}}$$

$\bar{F}_1 = 25.71i + 8.57j + 12.857k$ (kN)



$\bar{F}_2 = 20 \cos 60 \, i - 20 \sin 60 k$

$\bar{F}_2 = 10i - 17.32k$ (kN)

$\boxed{\Sigma \bar{F} = 35.71i + 8.57j - 4.463k \text{ (kN)}}$

The moment about O, $\Sigma \bar{M}_o = \bar{r}_{OA} \times \bar{F}_1 + \bar{r}_{OB} \times F_2$

(Here $\bar{r}_{OA} \times \bar{F}_1$) is zero because F_1 passes thro' O.

$\Sigma \bar{M}_o = \bar{r}_{OB} \times F_2$

$$= \begin{vmatrix} i & j & k \\ 6 & 2 & 0 \\ 10 & 0 & -17.32 \end{vmatrix}$$

$= i(-34.64) - j(-103.92) + k(-20)$

$\boxed{\Sigma \bar{M}_o = -34.64i + 103.92j - 20k \text{ kNm}}$

Equilibrium of Non concurrent Space force system

[3D Rigid Body]

If a body is said to be equilibrium.

$\Sigma \bar{R} = 0, \quad \Sigma \bar{M} = 0$

if $\Sigma \bar{R} = 0 \ldots\ldots \Sigma F_x i + \Sigma F_y j + \Sigma F_z k = 0$ and

$\Sigma \bar{M} = 0 \ldots\ldots \Sigma M_x i + \Sigma M_y j + \Sigma M_z k = 0$

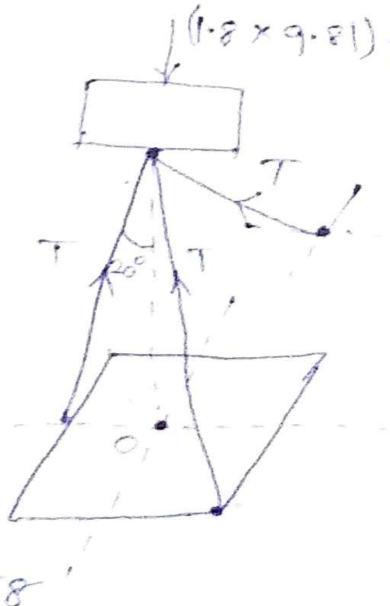
From these equation.

$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0$

$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0$

These are the six conditions for equilibrium of a rigid body subjected to non-concurrent space fo

1. A camara having mass of 1.8 kg rests on a tripod with legs equally spaced and each making an angle of 20°. Assuming the camara is at a point 1200 mm above ground level. Determine the force in eac



Let us assume that the compression reaction at each leg is T. Angle between the leg and vertical is given θ as 20°.

This system is considered as a concurrent force system. Applying equilibrium condition in y direction.

$\Sigma F_y = 0 \ldots\ldots 3T \cos 20 = 17.658$

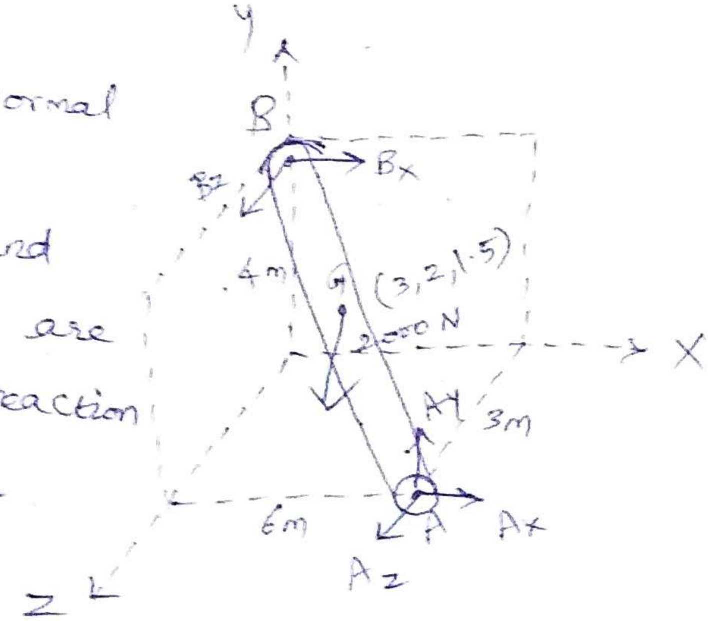
$\boxed{T = 6.26375 \ N}$

A bar of uniform cross section and homogeneous weighing 2000N is having a ball and socket support at 'A', and its end 'B' is resting at the corner of smooth walls as shown in fig. Determine the reactions developed at the ends A & B.

Soln:-

B_x, B_z are the normal reactions at 'B'.

At 'A', Ball and Socket joint, there are 3 components of reaction A_x, A_y and A_z



The Co-ordinates $A\,[6, 0, +3]$

$B\,[0, 4, 0]$

$G\,[3, 2, 1\cdot 5]$

~~Distance vector~~ 2 reactions, R_A, R_B & 1 load W.

There are three forces given by vectors.

$$\overline{R_B} = B_x i + B_z k \quad \text{——} (1)$$

$$\overline{W} = -2000\, j \ (N) \quad \text{——} (2)$$

$$\overline{R_A} = A_x i + A_y j + A_z k \quad \text{——} (3)$$

There are 5 unknown reactions.

Moment about A, $\Sigma \overline{M}_A = \overline{r}_{AB} \times \overline{R}_B + \overline{r}_{AG} \times \overline{W}$

Distance vector, AB, $\overline{r}_{AB} = -6i + 4j - 3k$

AG, $\overline{r}_{AG} = -3i + 2j - 1.5k$

So, $$\Sigma \overline{M}_A = \begin{vmatrix} i & j & k \\ -6 & 4 & -3 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} i & j & k \\ -3 & 2 & -1.5 \\ 0 & -2000 & 0 \end{vmatrix}$$

$$= i(4B_z) - j(-6B_z + 3B_x) + k(-4B_x)$$

$$+$$

$$i(-3000) - j(0) + k(6000)$$

$$\Sigma \overline{M}_A = (4B_z - 3000)i + (6B_z - 3B_x)j + (6000 - 4B_x)$$

$$\Sigma \overline{M}_A = M_x i + M_y j + M_z k$$

[equilibrium condns]

$\Sigma M_x = 0, \Sigma M_y = 0 \ \& \ \Sigma M_z = 0$

$\Sigma M_x = 0 \Rightarrow 4B_z - 3000 = 0 \Rightarrow$ $\boxed{B_z = 750\ N}$

$\Sigma M_y = 0 \Rightarrow 6B_z - 3B_x = 0 \Rightarrow$ $\boxed{B_x = 1500\ N}$

$\overline{R}_B = 1500i + 750k$ [from eqn (1)]

Now, Resultant force = Reaction at A & B + Load W.

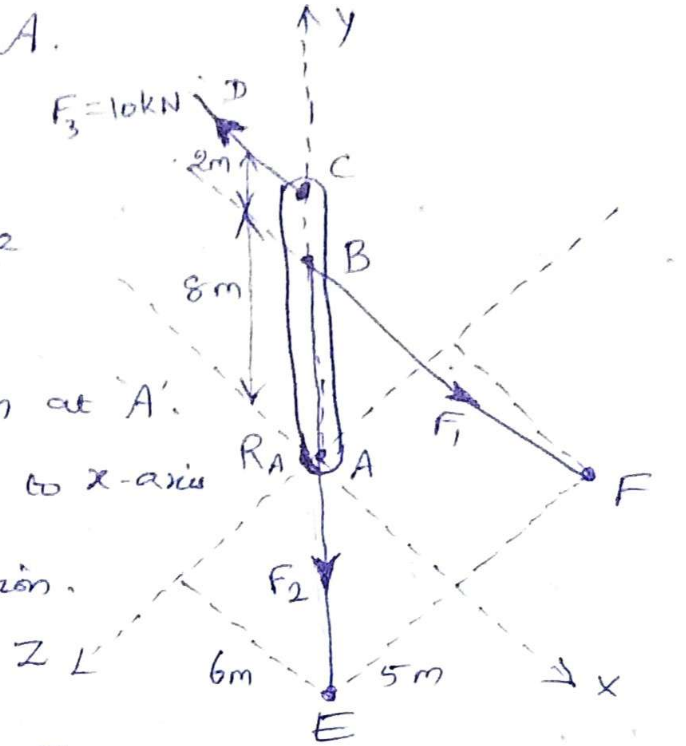
= (1) + (2) + (3)

$$\Sigma \overline{R} = (1500 + A_x)i + (-2000 + A_y)j + (750 + A_z)$$

$$\Sigma \overline{R} = \Sigma F_x i + \Sigma F_y j + \Sigma F_z k$$

$\Sigma F_x = 0, \Sigma F_y = 0, \Sigma F_z = 0,$ $\boxed{A_x = -1500\ N, A_y = 2000\ N, A_z = -750}$

6. A 10m pole supports a horizontal rope CD and is held by a ball and socket at A and two ropes BE & BF as shown in fig. If the tension in rope CD is 10kN and assuming that CD is ||l to the x axis, determine the tension in ropes BE and BF and the reaction at A.



Let F_1 & F_2 be the forces at the cable BF, BE.

R_A be the reaction at 'A'.

CD is the force ||l to x-axis

So, 3 forces, 1 reaction.

The co-ordinates of

A [0,0,0]

B [0,8,0]

C [0,10,0]

E [6,0,5]

F [6,0,-5]

3 forces, and 1 reaction can be written in vector form.

$$\overline{F_3} = -10i \text{ (kN)}$$

$$\overline{R_A} = A_x i + A_y j + A_z k \quad \text{(Ball \& Socket support)}$$

$$F_1 = \frac{F_1(\overline{BF})}{|BF|} = F_1 \frac{(6i - 8j - 5k)}{\sqrt{6^2+8^2+5^2}} = 0.536F_1 i - 0.715F_1 j - 0.447F_1 k$$

$$F_2 = \frac{F_2(\overline{BE})}{|\overline{BE}|} = F_2 \frac{(6i - 8j + 5k)}{\sqrt{}} = 0.536F_2 i - 0.715F_2 j + 0.447F_2 k$$

Equilibrium Conditions:

$\Sigma R = 0$, $\Sigma F_x, \Sigma F_y, \& \Sigma F_z = 0$

$\Sigma M = 0$, $\Sigma M_x, \Sigma M_y \& \Sigma M_z = 0$.

Taking Moment about 'A'.

$$\Sigma \bar{M}_A = \bar{r}_{AC} \times \bar{F}_3 + \bar{r}_{AB} \times \bar{F}_1 + \bar{r}_{AB} \times \bar{F}_2$$

$$\bar{r}_{AC} = 10\bar{j} \quad ; \quad \bar{r}_{AB} = 8\bar{j}$$

$$\Sigma \bar{M}_A = \cancel{10j \times (-10i)} \begin{vmatrix} i & j & k \\ 0 & 10 & 0 \\ -10 & 0 & 0 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 8 & 0 \\ 0.536F_1 & -0.715F_1 & -0.447F_1 \end{vmatrix} + \begin{vmatrix} i & j & k \\ 0 & 8 & 0 \\ 0.536F_2 & -0.715F_2 & 0.\ldots \end{vmatrix}$$

$$= \{k(100)\} + \{i(-3.576F_1) - j(0) + k(-\overset{4.288}{\cancel{3.576}}F_1)\}$$

$$+ \{i(3.576F_2) - j(0) + k(-\overset{4.288F_2}{\cancel{3.576F_2}})\}$$

$$\Sigma \bar{M}_A = i(3.576F_2 - 3.576F_1) + 0j + k(100 - \overset{4.288}{\cancel{3.576}}F_1 - \overset{4.288}{\cancel{3.5\ldots}}$$

$$\Sigma \bar{M}_A = \Sigma M_x i + \Sigma M_y j + \Sigma M_z k$$

$\Sigma M_x = 0$, $3.576F_2 - 3.576F_1 = 0 \Rightarrow F_1 = F_2$ —— (1)

$\Sigma M_z = 0$, $100 - 3.576F_1 - 3.576F_2 = 0 \Rightarrow \overset{4.288}{\cancel{3.576}}(F_1 + F_2) = 100$ —— (2)

From eqn (2) $\Rightarrow$ $4.288(x) = 100$ [$\because$ Put $F_1 + F_2 = x$]

$x = 23.32$

$F_1 + F_2 = 23.32\,kN$ —— (3)

~~from eqn (1)~~ $\Rightarrow$ Subs eqn (1) in (3)

$2F_1 = 23.32$, so, $\boxed{F_1 = F_2 = 11.66\,kN}$

$\Sigma \bar{R} = \bar{F}_3 + \bar{R}_A + \bar{F}_1 + \bar{F}_2$

$\Sigma \bar{R} = i(-10 + A_x + 0.536F_1 + 0.536F_2) + j(A_y - 0.715F_1 - 0.715F_2)$

$+ k(A_z - 0.447F_1 + 0.447F_2)$

$\Sigma \bar{R} = \Sigma F_x i + \Sigma F_y j + \Sigma F_z k$

$\Sigma F_x = 0, \quad -10 + A_x + 0.536F_1 + 0.536F_2 = 0$

$$\boxed{A_x = -2.49952 \text{ kN}}$$

$\Sigma F_y = 0, \quad A_y - 0.715F_1 - 0.715F_2 = 0$

$$\boxed{A_y = 16.6738 \text{ kN}}$$

$\Sigma F_z = 0, \quad A_z - \cancel{0.447F_1} + \cancel{0.447F_2} = 0 \quad [\because F_1 = F_2]$

$$\boxed{A_z = 0}$$

Answers:

1. Tensions in the ropes, $F_1 = F_2 = 11.66$ kN

2. Reactions at A is gn by, $R_A = (-2.49952i + 16.6738j)$ kN

Centroid:

Centroid is defined as the point at which the total area, Volume, or length is assumed to be Concentrated. It is related to distribution of length, area and Volume.

Centre of Mass:

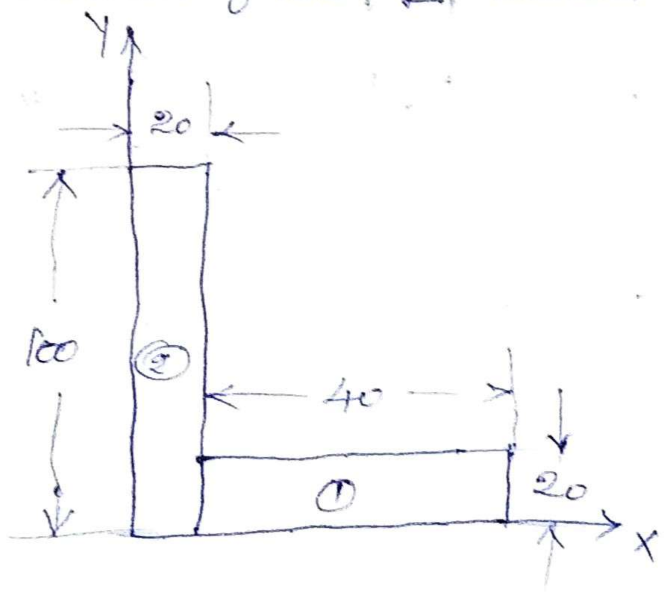
Centre of Mass is a point where the entire mass of a body may be assumed to be Concentrated.

Centre of Gravity:

It is a point through which the line of action of the weight of the body passes irrespective of the position of the body. It is represented as C.G. It is related to distribution of mass.

Numerical Problems:

1. Find the centroid of the, L, section Shown in fig.



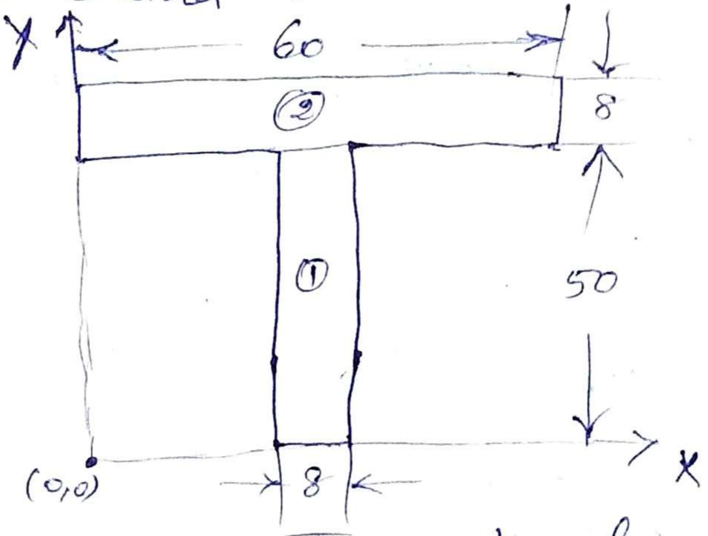
S.No	Section	Area (mm^2)	$\bar{x}$ (mm)	$\bar{y}$ mm	$A\bar{x}$	$A\bar{y}$
1.	Rectangle	40×20	$20 + \frac{40}{2}$	$20/2$	32000	8000
2.	Rectangle	100×20	$\frac{20}{2}$	$100/2$	20000	100000
		$\Sigma A = 2800$			$\Sigma A\bar{x} = 52000$	$\Sigma A\bar{y} = 108000$

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{52000}{2800} = 18.571 \text{ mm}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{108000}{2800} = 38.57 \text{ mm}$$

Centroid $C\,[18.571,\ 38.57]$ mm

2. Find the centroid of the T-section shown in fig.



This section is symmetrical about Y-axis

So $\bar{x} = \frac{60}{2} = 30$ mm

Because of symmetry $\bar{X}$ can be located immediately

$$\bar{X} = \frac{60}{2} = 30 \text{ mm}$$

S.No	Section	A (mm^2)	$\bar{Y}$ (mm)	$A\bar{y}$
1.	Rectangle	50×8	$50/2$	10000
2.	Rectangle	60×8	$50 + \left(\frac{8}{2}\right)$	25920
		$\Sigma A = 880$		$\Sigma A\bar{y} = 35920$

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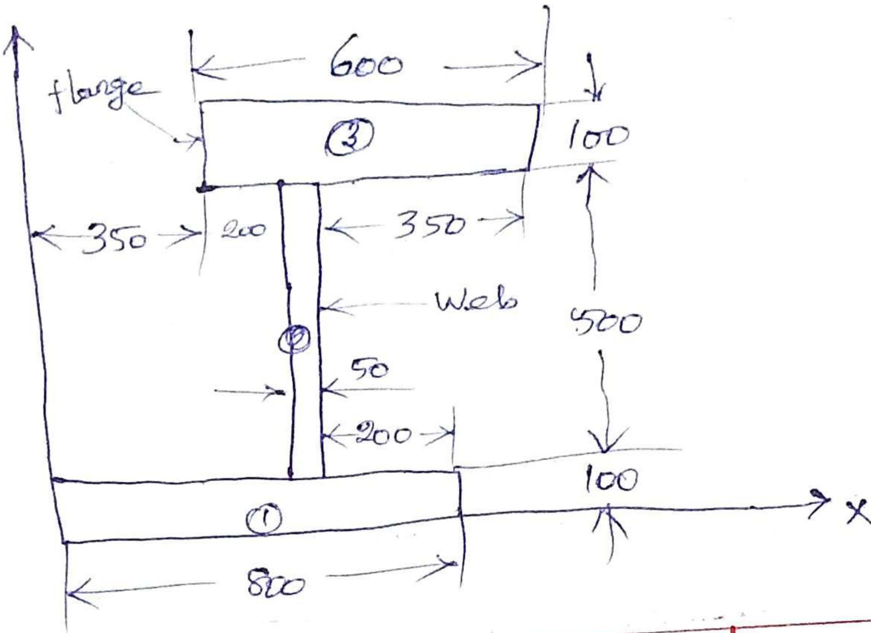
$$\bar{Y} = \frac{\Sigma A \bar{y}}{\Sigma A} = \frac{35920}{880} = 40.818 \text{ mm}$$

$\therefore$ Centroid, $[30, 40.818]$ mm

Q. Locate the centroid of the I section shown in fig.



S. No	Section	A (mm²)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$A\bar{x}$	$A\bar{y}$
1.	Rectangle	800×100	800/2	100/2	32×10^6	4×10^6
2.	Rectangle	500×50	$550+\frac{50}{2}$	$100+\frac{500}{2}$	14.375×10^6	8.75×10^6
3.	Rectangle	600×100	$350+\frac{600}{2}$	$600+\frac{100}{2}$	39×10^6	39×10^6
		165×10^3			85.375×10^6	51.75×10^6

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = 517.42 \text{ mm}$$

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = 313.64 \text{ mm}$$

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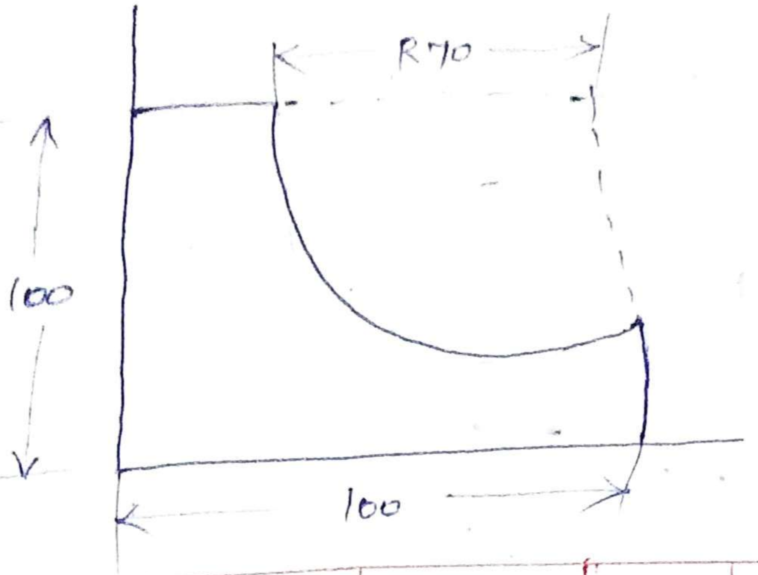
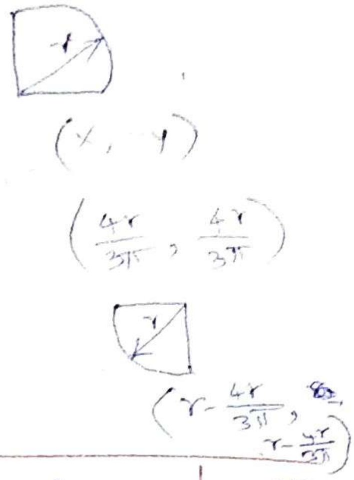
4. Locate the centroid of the area shown in fig.





S.No	Section	Area	$\bar{x}$	$\bar{y}$	$A\bar{x}$	$A\bar{y}$
1.	Square	100x100	100/2	100/2	500×10^3	500×10^3
2.	Quarter circle	$\frac{\pi r^2}{4}$ $=-3848.45$	$30+70-\frac{4(70)}{3\pi}$ $=70-29$	70-29	-270.507×10^3	-270.5
		$=6151.55$			$=229493$	$=229$

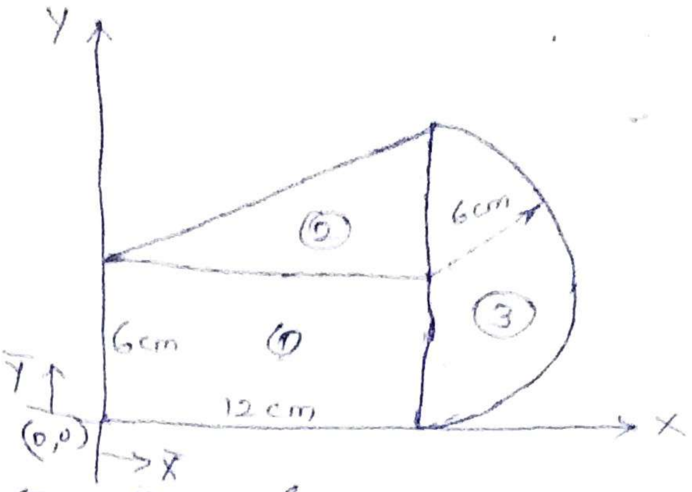
$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = \frac{229493}{6151.55} = 37.3065 \text{ mm}$$

$$\frac{Ax_1 - Ax_2}{A_1 - A_2}$$

$$\bar{Y} = \frac{\Sigma(A\bar{Y})}{\Sigma A} = 37.3065 \text{ mm}$$

$$\frac{Ay_1 - Ay_2}{A_1 - A_2}$$

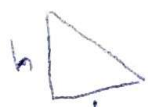
1. Locate the centroid of area shown in fig.



Soln: Area ①: Rectangle

$A_1 = 12 \times 6 = 72\ cm^2$

$x_1 = \frac{12}{2} = 6\ cm\ ,\quad y_1 = \frac{6}{2} = 3\ cm$



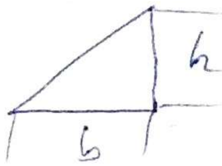
(x, y)

$\left(\frac{b}{3}, \frac{h}{3}\right)$

Area ②: Right angle Triangle

$A_2 = \frac{1}{2} bh = \frac{1}{2}(12)(6) = 36\ cm^2$

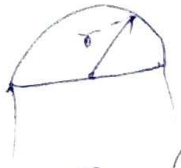
$x_2 = b - \frac{b}{3} = 12 - \frac{\cancel{12}^{4}}{\cancel{3}} = 8\ cm$



$y_2 = 6 + \frac{h}{3} = 6 + \frac{6}{3} = 8\ cm$

Area ③: Semicircle

$A_3 = \frac{\pi r^2}{2} = \frac{\pi (6)^2}{2} = 56.55\ cm^2$



$x_3 = 12 + \frac{4r}{3\pi} = 14.546\ cm$

$C = \left(\frac{d}{2}, \frac{4R}{3\pi}\right)$

$y_3 = \frac{12}{2} = \frac{d}{2} = 6\ cm$

$\overline{X} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{\Sigma A} = \frac{1542.5763}{164.55} = 9.374\ cm$

$C = \left(\frac{4r}{3\pi}, \frac{d}{2}\right)$

$\overline{Y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{\Sigma A} = \frac{\overset{843.3}{\cancel{627.3}}}{164.55} = 5.125\ cm$

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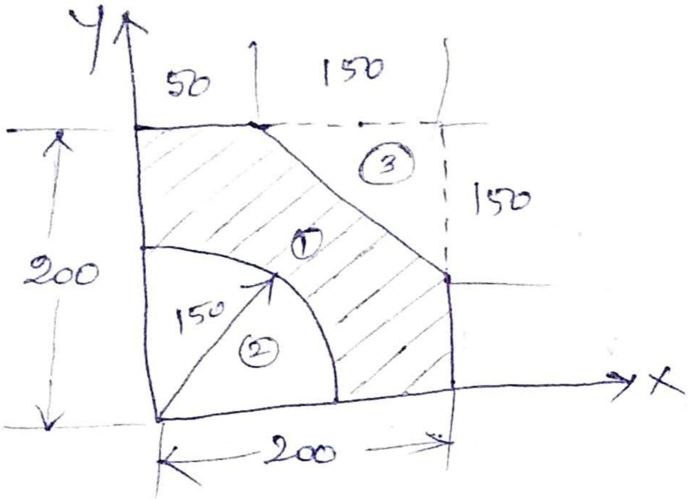
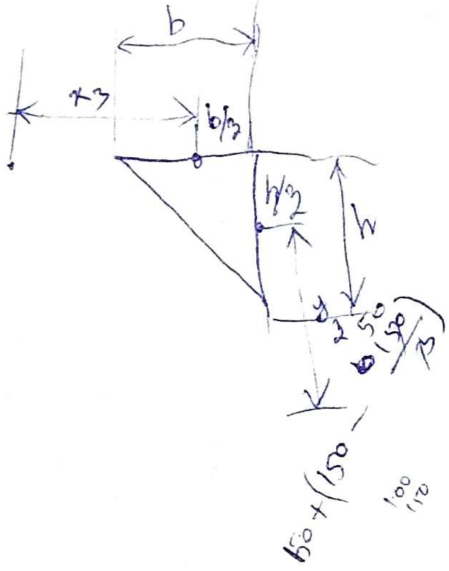
8. Locate the centroid of the plane shaded area shown in fig





Area-① Square

$A_1 = 200 \times 200 = 40000\,mm^2$

$x_1 = 100\ mm, \quad y_1 = 100\ mm$

Area-② Quarter circle

$A_2 = \frac{\pi r^2}{4} = 17671.46\,mm^2$

$x_2 = \frac{4R}{3\pi} = 63.66\ mm$

$y_2 = \frac{4R}{3\pi} = 63.66\ mm$

Area-③ Right angle Triangle

$A_3 = \frac{1}{2} bh = \frac{1}{2} \times 150 \times 150 = 11250\,mm^2$

$x_3 = 200 - \frac{b}{3} = 200 - \frac{150}{3} = 150\,mm \quad \left\{50 + \left(150 - \frac{b}{3}\right)\right\}$

$y_3 = 50 + 150 - \frac{\cancel{150}^{50}}{\cancel{3}} = 150\ mm$

$$\bar{X} = \frac{A_1x_1 - A_2x_2 - A_3x_3}{A_1 - A_2 - A_3} = \frac{1.1875 \times 10^6}{\cancel{68921.46}\ 11078.54} = 107.189\ mm$$

$$\bar{Y} = \frac{A_1y_1 - A_2y_2 - A_3y_3}{A_1 - A_2 - A_3} = \frac{1.1875 \times 10^6}{11078.54} = 107.189\ mm$$

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Area Moment of Inertia:

The property which gives the measure of resistance to bending in the case of plane area or plates is known as Moment of Inertia of the area.

Mass Moment of Inertia:

For 3 dimensional rigid bodies the resistance to rotation is measured by mass moment of Inertia.

Radius of Gyration:

Radius of gyration is the effective distance where the entire area/Mass may be considered to be located

$$k_x = \sqrt{\frac{I_{xx}}{A}} \; ; \; k_y = \sqrt{\frac{I_{yy}}{A}}$$ in the case of plane sections

Parallel axis Theorem:

Moment of inertia of a plane area which is ||l to centroidal axis about an axis is equal to the sum of moment of inertia about centroidal axis and product of area and square of the distance between the two ||l axes. $I_{AB} = I_g + Ad^2$

MI of an about its centre of mass

d is distance between two axis

Perpendicular axis Theorem/Polar Moment of Inertia:

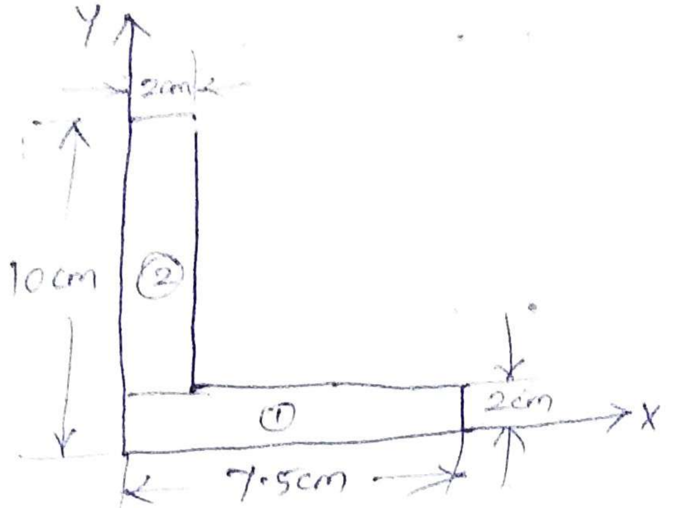
Moment of Inertia about an axis ⊥r to the plane of an area is known as polar moment of Inertia. It is denoted by I_{ZZ}. Here z axis is ⊥r to both centroidal x-axis and y-axis and passing through the centre.

$$I_p = I_{zz} = I_{xx} + I_{yy}$$

The above eqn is called as ⊥r axis theorem.

M.I about an ⊥r axis = Sum of M.I about any two ⊥r axis through the same point lying the same plane.

1. Calculate moment of Inertia of L section about horizontal and vertical axis passing through centroid axis. Also find the radius of gyration about centroid axis.



S.No	Section	Area (cm^2)	$\bar{x}$	$\bar{y}$	$A\bar{x}$	$A\bar{y}$
1.	Rectangle	7.5×2	$7.5/2$	$2/2$	56.25	15
2.	Rectangle	8×2	$2/2$	$2+\frac{8}{2}$	16.00	96
		31			72.25	111

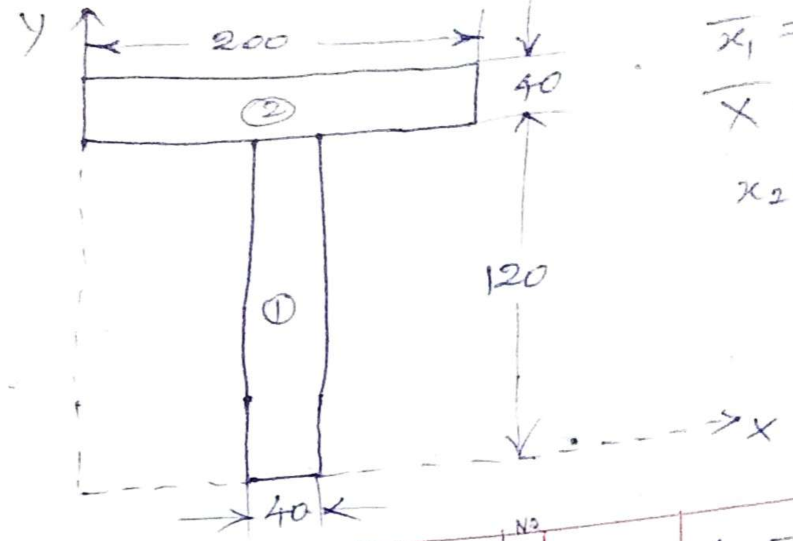
$\bar{X} = 2.33\text{ cm}$, $\bar{Y} = 3.58\text{ cm}$.

No	I_{xxg}	$dx=\bar{y}-\bar{Y}$	$A(dx)^2$	$I_{xx}=I_{xxg}+A(dx)^2$	I_{yyg}	$dy=\bar{x}-\bar{X}$	$A(dy)^2$	$I_{yy}=I_{yyg}+A(dy)^2$
1	$\frac{7.5\times2^3}{12}$	-2.58	99.846	104.846	$\frac{2\times7.5^3}{12}$	1.42	30.246	100.558
2	$\frac{2\times8^3}{12}$	2.42	93.7024	179.035	$\frac{8\times2^3}{12}$	-1.33	28.3024	33.6357

$I_{xx} = 283.881\ cm^4$ $\quad$ $I_{yy} = 134.1935\ cm^4$

$$K_{xx} = \sqrt{\frac{I_{xx}}{\Sigma A}} = 3.026\text{ cm};\quad K_{yy} = \sqrt{\frac{I_{yy}}{\Sigma A}} = 2.08\text{ cm}.$$

2. Determine the moment of Inertia and radius of gyration of T Section about centroidal y axis.



$\bar{x}_1 = 10\,cm$

$\bar{X} = 10\,cm$ (symmetric)

$x_2 = 10\,cm$

S.No.	Section	Area (cm^2)	$\bar{x}$ (cm)	$A\bar{x}$	I_{yyg}	$dy = \bar{x} - \bar{X}$	$A(dy)^2$	I_{yy}
1.	Rectangle	4×12	$8+\frac{4}{2}$	480	$\frac{12\times4^3}{12}$	0	0	64
2.	Rectangle	20×4	$20/2$	800	$\frac{4\times20^3}{12}$	0	0	2666.6
		128		1280				2730

Answers:

$$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} = 10\ cm$$

$$I_{yy} = 2730.667\ cm^4$$

Radius of gyration about y axis, $k = \sqrt{\frac{I_{yy}}{\Sigma A}}$

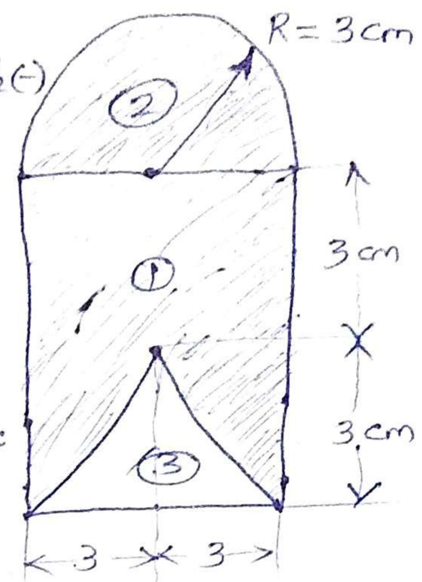
$= 4.168\ cm$

$= 41.68\ mm$

6. Compute the second moment of area of the plane surface shown in fig. about its horizontal centroidal axe

Soln :-

The section involves:



1. a square (+), 2. a semicircle (+), 3. a triangle (−)

$A_1 = 36 cm^2$

$A_2 = \frac{\pi \times 3^2}{2} = 14.137\ cm^2$

$A_3 = \frac{1}{2} bh = \frac{1}{2}(6)(3) = 9\ cm^2$

Then, centres of individual areas are calculated from the base as follows:

$y_1 = 3\ cm$

$y_2 = 6 + \frac{4(3)}{3\pi} = 7.27\ cm$

$y_3 = \frac{h}{3} = 1\ cm.$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 - A_3 y_3}{\Sigma A} = \frac{36(3) + 14.137(7.27) - 9(1)}{36 + 14.137 - 9}$$

$\bar{y} = 4.96 cm$ from the base.

$dx_1 = y_1 - \bar{y} = -1.906\ cm$

$dx_2 = y_2 - \bar{y} = 2.364\ cm$

$dx_3 = y_3 - \bar{y} = -3.906\ cm$

$$I_{xx} = \left[I_{xxg} + A_1(dx_1)^2\right] + \left[I_{xxg} + A_2(dx_2)^2\right] - \left[I_{xxg} + A_3(dx_3)^2\right]$$

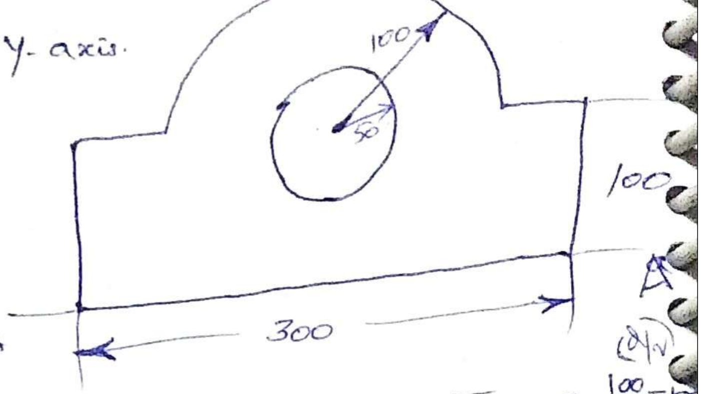
$$= \left[\frac{6 \times 6^3}{12} + (36) \times 1.906^2\right] + \left[\overset{0.11(R^4)}{0.11(3)^4} + 14.137(2.364)^2\right] - \left[\overset{bh^3/36}{\frac{6 \times 3^3}{36}} + 9(3.906)^2\right]$$

$$= 238.78 + \overset{110.8}{\cancel{87.91}} - 141.81$$

$I_{xx} = \cancel{184.8784}\ cm^4$

207.77

7. Determine the Second Moment of area of the Section shown in fig about its base axis a-a.



Object is Symmetrical about Y-axis.

So, $\bar{X} = \frac{300}{2} = 150$ mm

Find $\bar{Y}$:

$A_1 = 300 \times 100 = 30000$ mm^2

$A_2 = \frac{\pi R^2}{2} = \frac{\pi (100)^2}{2} = 15707.96$ A

$A_3 = \pi R^2 = 7853.98$ mm^2

$\bar{y}_1 = \frac{100}{2} = 50$ mm ; $\bar{y}_2 = 100 + \frac{4(100)}{3\pi} = 142.44$ mm ; $\bar{y}_3 = 50 + \frac{100}{2} = 100$

$$\bar{Y} = \frac{(30000 \times 50) + (15707.96 \times 142.44) - (7853.98 \times 100)}{30000 + 15707.96 - 7853.98} = 77.98 \text{ mm}$$

S.No	Section	A	$\bar{y}$	$A\bar{y}$	I_{xxg}	$d_x = \bar{y} - \bar{Y}$	$A(d_x)^2$	I_{xx}
1.	Rectangle	30000	50	1.5×10^6	$\frac{300 \times 100^3}{12}$	-27.98	23.4864×10^6	48.49
2.	Semicircle	15707.96	142.44	2.24×10^6	$0.11(100)^4$	64.46	65.268×10^6	104.54× 76.268
3.	Circle	-7853.98	100	-785398	$\frac{\pi (50)^4}{4}$	22.02	3.808×10^6	$-8.72 \times$
		37853.98		2.95×10^6			$I_{xx} =$	$116.038 \times$

From ∥l axis theorem, $I_{AA} = I_{xx} + \Sigma A(\text{dist})^2$; dist $= \bar{Y}$

$$I_{AA} = 116.038 \times 10^6 + (37853.98 \times 77.98^2)$$

$$= 346.223 \times 10^6 \text{ mm}^4$$

374.49×10^6

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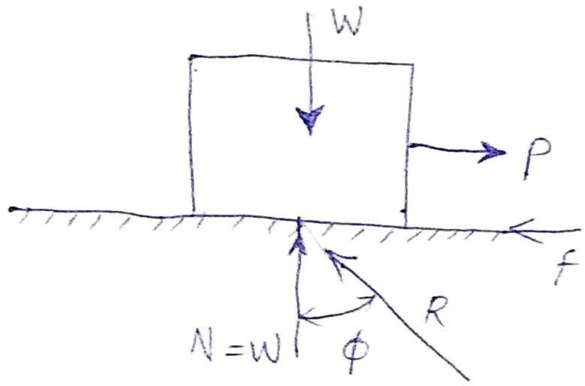
Part - 1 : Friction and its Applications

Definition:-

Friction is the contact resistance developed between the contact surfaces, when one of the body moves or tends to move over the other. It always opposes the motion or tends to move the body.



Angle of friction: (ϕ) :-

The angle ϕ is called angle of friction which is defined as the angle between the normal force (N) and the frictional resultant (R). 'ϕ' depends on the nature of two surfaces in contact.

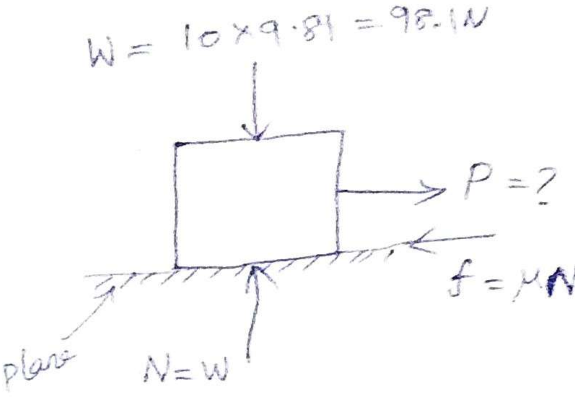
Co-efficient of friction (μ) :-

The ratio between frictional force (f) and the normal force (N) is called co-efficient of friction (μ).

$$\boxed{\tan\phi = \frac{f}{N} = \mu} \Rightarrow \boxed{f = \mu N}$$

It depends on the nature of the material surface.

① A block of 10 kg is kept on a horizontal plane. Find the force required to cause motion, if the force applied is ||l to the plane. Take the coefficient of friction is 0.25



Soln:-

Apply equilibrium conditions,

$\underline{\Sigma F_y = 0}$ .... $N = 98.1\ N$

$\underline{\Sigma F_x = 0}$ ... $P = f$

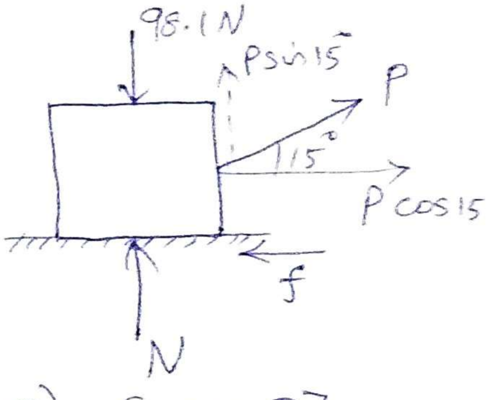
$= \mu N$

$= 0.25 \times 98.1$

$\boxed{P = 24.525\ N}$

---

② A block of 10 kg is kept on a horizontal plane. Find the force required to cause motion, if the applied force is 15° with the horizontal plane. Take the coefficient of friction is 0.25.



Soln:- Apply equilibrium Condns,

$\underline{\Sigma F_y = 0}$, $N + P\sin 15 = 98.1N$

$N = -P\sin 15 + 98.1$ —— ①

$\underline{\Sigma F_x = 0}$, $f = P\cos 15$

$P\cos 15 = \mu N$

$P\cos 15 = 0.25\,(98.1 - P\sin\theta)$ [∴ eqn ①]

$P\cos 15 + (0.25 \times P\sin 15) = 0.25 \times 98.1$

$P\left[\cos 15 + 0.25\sin 15\right] = 0.25 \times 98.1$

$\boxed{P = 23.796\ N}$

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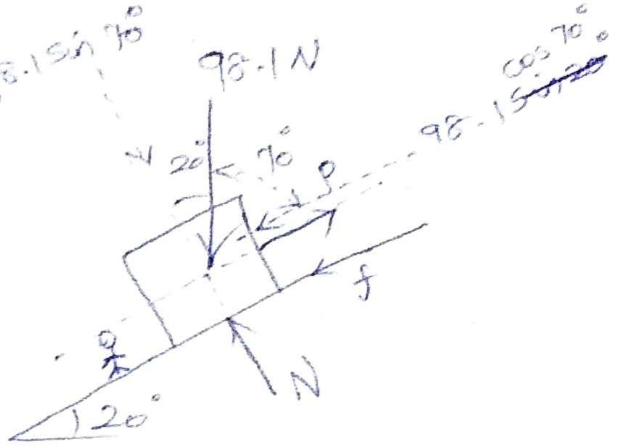
3. A body of weight 10 kg is placed on a rough inclined plane, which is 20° with horizontal. What is the minimum force required to raise the block if the applied force is ||l to the plane. The coefficient of friction $\mu = 0.25$

Apply equilibrium condns,



$\underline{\Sigma F_y = 0}$

$$N = 98.1 \sin 70$$

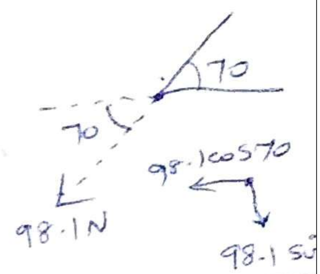
$$N = 92.1838\ N$$

$\underline{\Sigma F_x = 0}$

$$P = f + 98.1 \cos 70$$

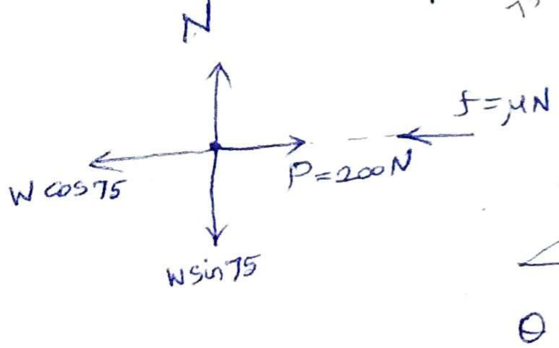
$$P = (0.25 \times 92.1838) + 98.1 \cos 70°$$

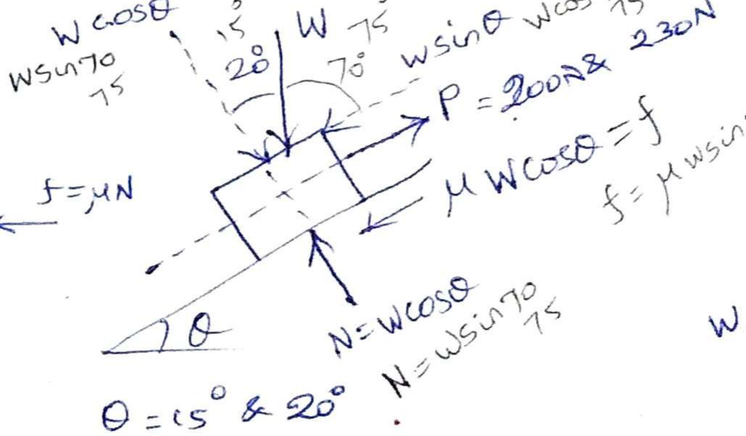
$$\boxed{P = 56.598\ N}$$



(5) An effort of 200 N is required to just move a certain body up on an inclined plane of 15°, the force acting parallel to plane.

If the plane angle $\theta = 20°$, the effort required again ||l to plane is found to be 230 N. Find weight of the body and coefficient of friction.

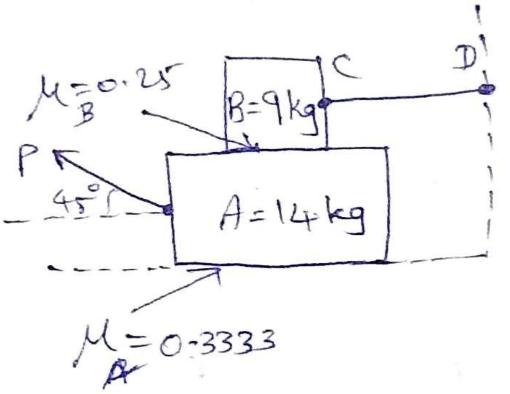


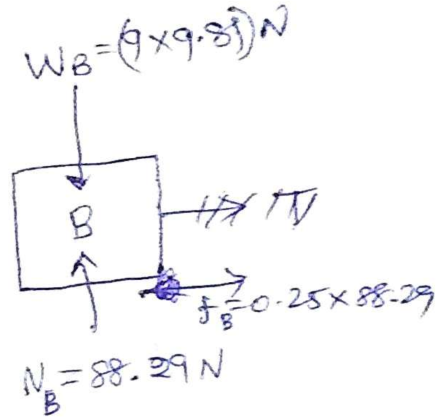


$\theta = 15°$ & $20°$

$W, \mu = ?$

7. Block B rest on block A = 14 kg and is attached by a horizontal rope CD to the wall as shown in fig. What force P is necessary to cause motion of A to start. Take weight of B = 9 kg, μ between block and floor is 1/3 and between blocks 1/4.





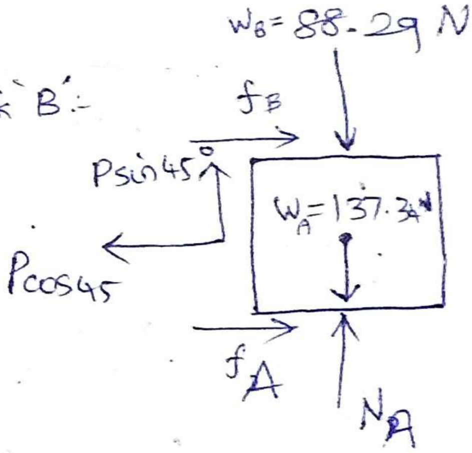
Applying equilibrium Condition in block 'B':-

$\Sigma F_y = 0$ .... $N_B = W_B = 88.29$ N

$\Sigma F_x = 0$ .... $f_B = \mu_B N_B$

$= 0.25 \times 88.29$

$f_1 = 22.0725$ N



Now Block 'A',

$\Sigma F_y = 0$ .... $N_A + P\sin45 = 88.29 + 137.34$

$N_A = 225.63 - P\sin45$

$f_A = \mu_A N_A$

$f_A = 0.3333\,[225.63 - P\sin45]$

$\Sigma F_x = 0 \ldots\ldots \quad P\cos 45 = f_1 + f_2$

$$P\cos 45 = 22.0725 + \{0.3333[225.63 - P\sin 45]\}$$

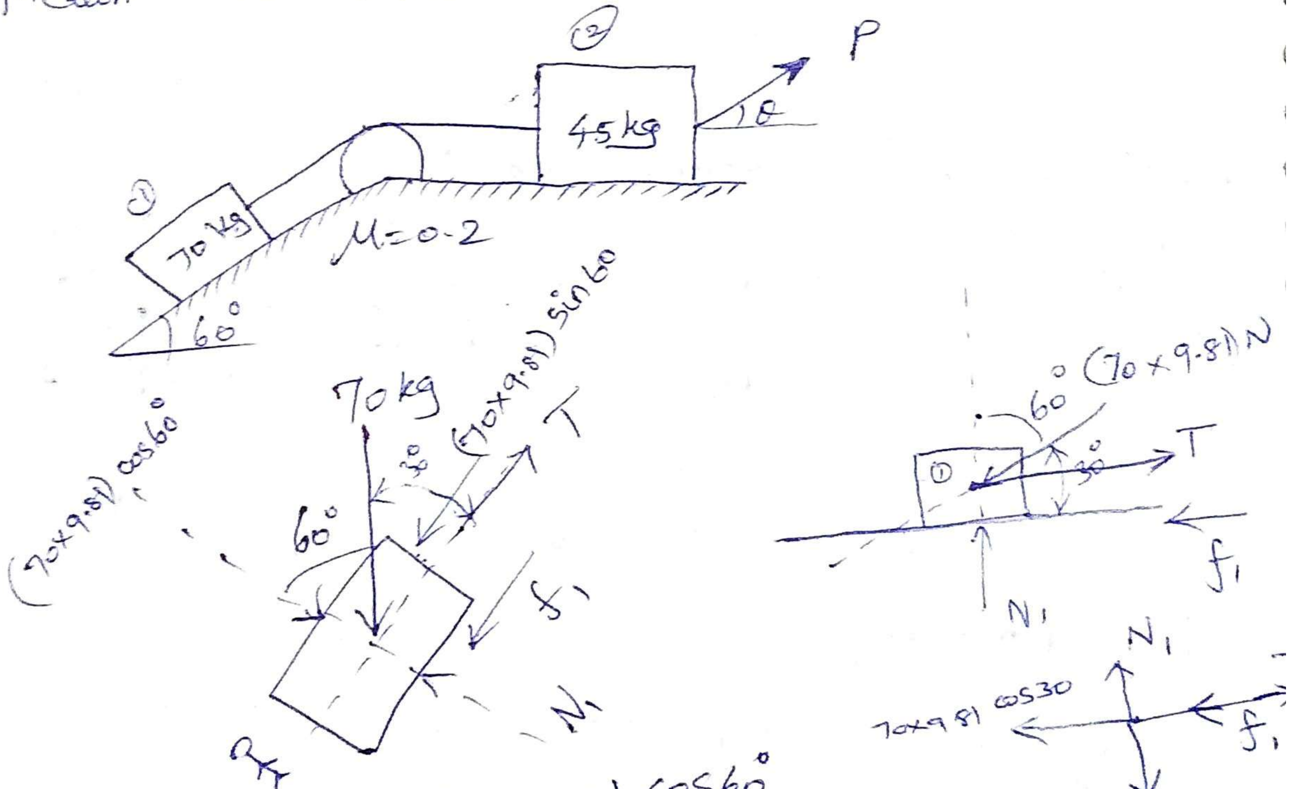
$$P\cos 45 + 0.2356\,P = 97.2749$$

$$P(0.9427) = 97.2749$$

$$\boxed{P = 103.1868\ N}$$

---

12. What is the least value of P to cause the motion to ~~impend~~ move? Assume coefficient of friction to be 0.20.



$\Sigma F_y = 0 \ldots\ldots \quad N_1 = (70\times 9.81)\cos 60°$

$f_1 = \mu N_1$

$f_1 = 0.2 \times (70\times 9.81)\cos 60° = 68.67\ N$

$\Sigma F_x = 0 \ldots\ldots \therefore \quad T = f_1 + (70\times 9.81)\sin 60$

$$\boxed{T = 663.3696\ N} \quad \text{— (1)}$$

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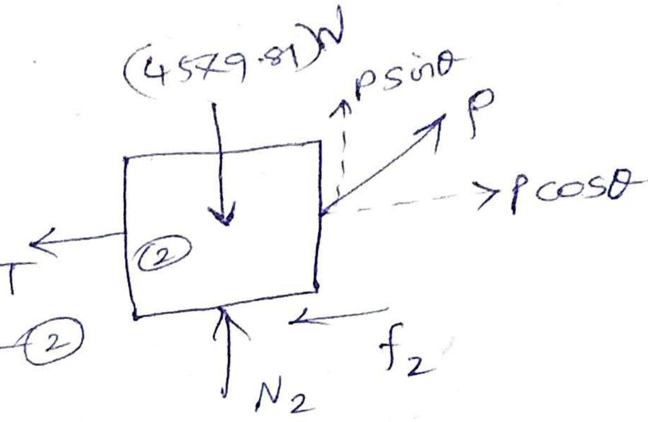
Consider 45 kg block :-



$\Sigma F_y = 0 \ldots \quad N_2 = -P\sin\theta + (45 \times 9.81)$

$f_2 = \mu N_2$

$f_2 = 0.2[-P\sin\theta + 441.45]$

$f_2 = 88.29 - 0.2P\sin\theta$ ——— ②

$\Sigma F_x = 0 \ldots \quad P\cos\theta = T + f_2$

Subs eqn ① & ②

$P\cos\theta = 663.369 + 88.29 - 0.2P\sin\theta$

$P\cos\theta + 0.2P\sin\theta = 751.659$

$P[\cos\theta + 0.2\sin\theta] = 751.659$

$$P = \frac{751.659}{\cos\theta + 0.2\sin\theta} \quad \text{——— ③}$$

When 'P' is least, the denominator $[\cos\theta + 0.2\sin\theta]$ must be maximum.

i.e., $\frac{d}{d\theta}[\cos\theta + 0.2\sin\theta] = 0$

$-\sin\theta + 0.2\cos\theta = 0$

$0.2 = \frac{\sin\theta}{\cos\theta}$

$0.2 = \tan\theta$

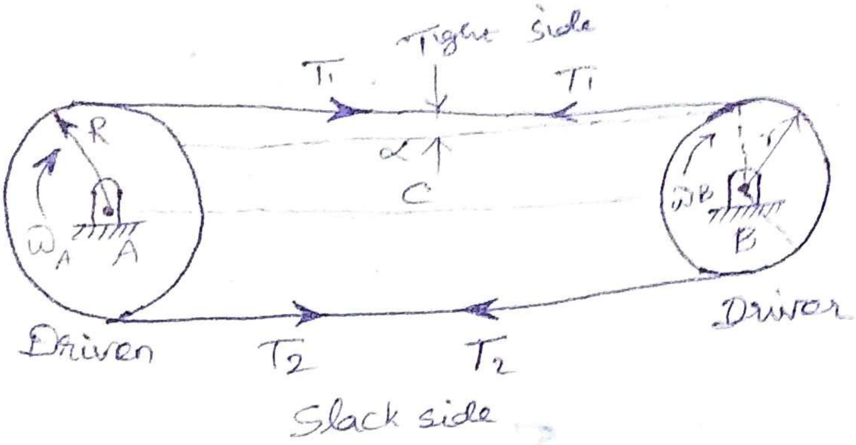
$\theta = 11.3099^\circ$

Subs in eqn ③, $\boxed{P = 737.0628\,N}$

Part - 2

Belt Friction

The frictional force exerted between the belt and the pulley contact surface is known as belt friction



The various types of belt are

i) Flat belt ii) V-Belt iii) Rope.

The ratio of tight side tension to slack side tension,

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

Where, μ - Co-efficient of friction between the belt and contact surface.

Always $T_1 > T_2$.

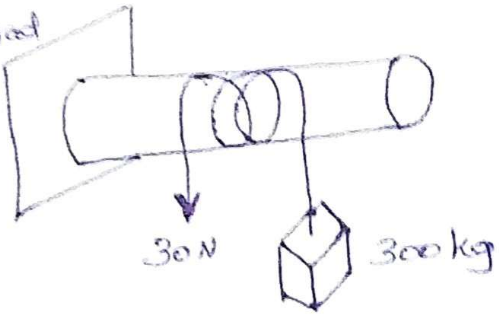
Flat belts are used, when the centre distance between the machinery is too high as in the case of rice mills, ~~flo mills~~, Stone crushers etc.

When the centre distance between driving and driven shaft is less as in the case of Lathes, grinders, and pumps, V-belts are used with the help of grooved pulley.

5.1.5

Numerical Problems:

① Three turns of rope around a horizontal post will hold a 300 kg mass with a pull of 30 N. Determine the co-efficient of friction between the rope and the post.



Angle of contact $= 3 \times 360^\circ \times \frac{\pi}{180}$ rad

$= 6\pi$ rad.

W.k.T $\quad \frac{T_1}{T_2} = e^{\mu\theta}$

$\ln(T_1/T_2) = \mu\theta$

$$\mu = \frac{\ln\left(\frac{300 \times 9.81}{30}\right)}{6\pi}$$

$$\boxed{\mu = 0.24329}$$

2. A belt embraces an angle of 200° over the surface of a pulley of 500 mm diameter. If the tight side tension of the belt is 2.5 kN, find out the slack side tension of the belt. $\mu = 0.$

Soln:- Angle of Contact $\theta = \frac{\pi}{180} \times 200^\circ = 3.49$ rad.

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.3 \times 3.49}$$

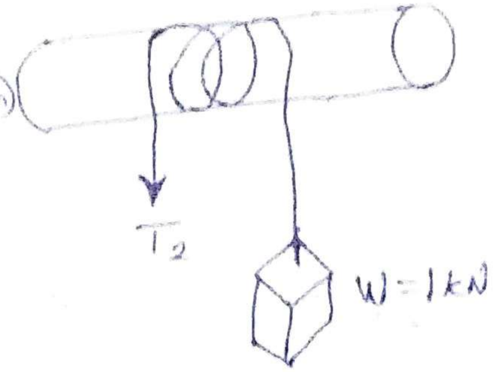
$$\frac{2.5}{T_2} = 2.84965$$

$$\boxed{T_2 = 0.8773 \text{ kN}}$$

8. A rope is wrapped three and a half times around a cylinder as shown in fig. Determine the force exerted on the free end of the rope that is required to support a 1 kN weight. The co-efficient of friction is $\mu = 0.25$.

Soln:

Assume the force required as slack side tension T_2 (min)



$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$\frac{1000}{T_2} = e^{0.25 \times 7\pi}$$

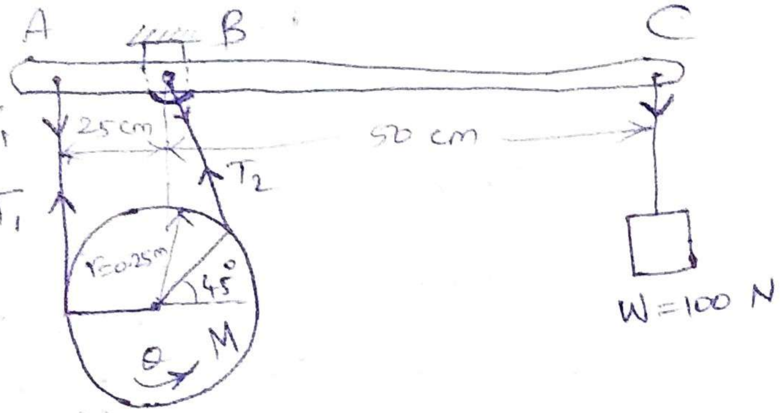
$$\boxed{T_2 = 4.0958 \text{ N}}$$

$3\frac{1}{2}$ turns $\Rightarrow \frac{\pi}{180} \times (3.5 \times 360)$

7π rad.

9. A cord is attached to a block of 50kg mass, the block is positioned on a 20° inclined plane, as shown in. The other end of the cord is supporting a cylinder. If the co-efficient of friction between block and inclined plane is 0.2 and co-eff. friction between cord and the cylindrical support surface is 0.3, determine the range of mass of cylinder for which the system is in equilibrium.

10. A single belt (Band) is used to brake a rotating wheel. The belt AB attached to a lever ABC hinged at B. The $\mu = 0.5$. Find the braking moment M exerted by a vertical weight W=100N



Soln:-

Assume T_1 & T_2 be the tight side and slack side tensions shown in fig.

$$\text{Angle of lap} = 180 + 45° = 225° = \frac{\pi}{180} \times 225 = 3.93 \text{ rad.}$$

$\mu = 0.5$,

From Pulley,

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

$$= e^{0.5 \times 3.93}$$

$$\frac{T_1}{T_2} = 7.12418 \quad \text{—— ①}$$

Consider lever,

Taking moment about 'B' and equating it to zero.

$$+(T_1 \times 25) - (100 \times 50) = 0$$

$$T_1 = 200 \text{ N}$$

$$T_2 = 28.073 \text{ N} \qquad [\because \text{eqn ①}]$$

Braking torque (or) Moment = applied moment

$$M = (T_1 - T_2)\, r_D$$

$$\boxed{M = 42.981 \text{ Nm}}$$

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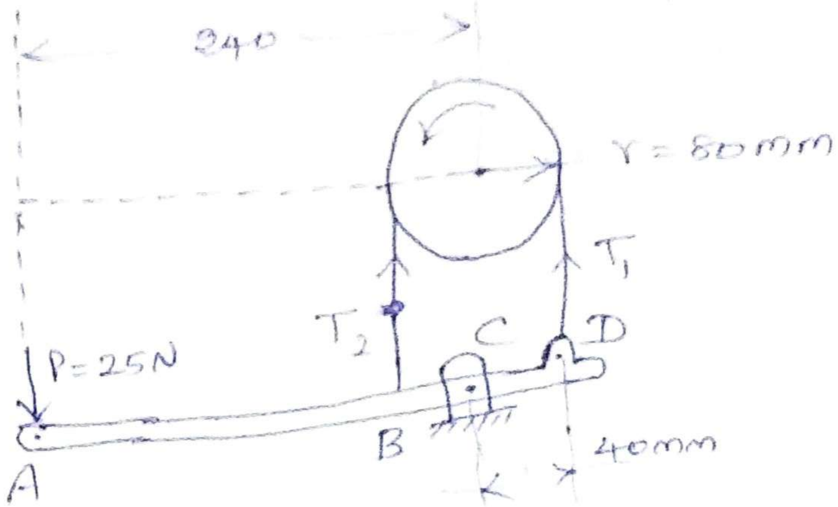
13. The speed of a brake drum is controlled by a belt attached to the lever AD as shown in fig. A force P of 25N is applied to the lever at A. Determine the magnitude of the couple applied to the drum, if the coefficient of friction between the belt and drum is 0.25. The drum is rotating CCW at a constant speed.



Consider brake drum,

contact angle $\theta = \pi$ rad.

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.25(\pi)}$$

$$\frac{T_1}{T_2} = 2.19328$$

$$T_1 = 2.19328\,T_2 \quad \text{——— (1)}$$

Consider lever AD,

Taking moment about hinge 'C'. and equating is zero,

$\Sigma M_C = 0.$

$$T_1 \times 40 + (25 \times 280) - (T_2 \times 120) = 0$$

$$40T_1 - 120T_2 = -7000$$

$$T_1 - 3T_2 = -175$$

$$T_1 = 3T_2 - 175 \quad \text{—— ②}$$

Solving eqn ① & ②,

① = ②

$$3T_2 - 175 = 2.19328\, T_2$$

$$T_2(3 - 2.19328) = 175$$

$$T_2 = 216.9278\, N$$

$$T_1 = 475.783\, N$$

Moment applied to the drum,

$$M = (T_1 - T_2)\, r_D$$

$$= (475.783 - 216.9278) \times 0.08$$

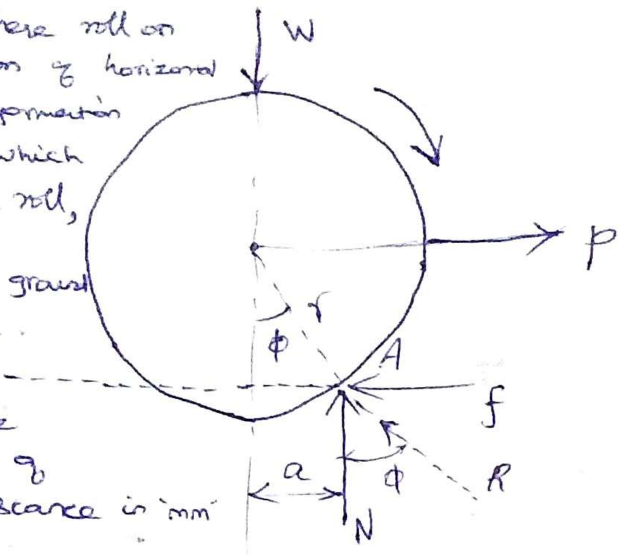
$$\boxed{M = 20.708\ Nm}$$

5-1-8

Wheel Friction, Rolling Resistance

hen a wheel (or) sphere roll on
ound under the action of horizontal
rce P there is a deformation
the surface upon which
wheel or sphere roll,
using the contact
etween wheel and ground
takes place over
ertain area.

R – Resultant force
a – co-efficient of rolling resistance in 'mm'



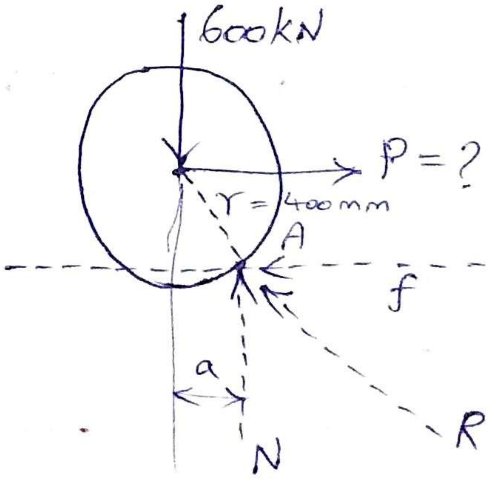
The value of co-efficient of rolling resistance $a = 0.25\,mm$ for steel wheel on steel rail.

Consider the above diagram, and taking moment about A, and equating it to zero for equilibrium

$\sum M_A = 0 \Rightarrow$ $\quad W r \sin\phi = P \times r$

---

1. A steel of 800 mm diameter rolls on a horizontal steel rail. It carries a load of 600 kN. The coefficient of rolling resistance is 0.25 mm. What is the force P required to roll the wheel along the rail.



$\sum M_A = 0$

$-P \times 400 + 600 \times a = 0$

$400P = 600(0.25)$

$P = \frac{600(0.25)}{400}$

$\boxed{P = 0.375\,kN}$

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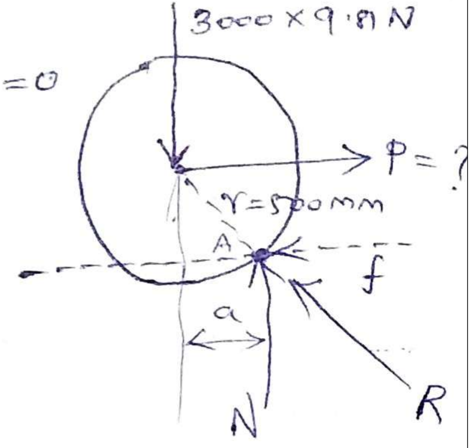
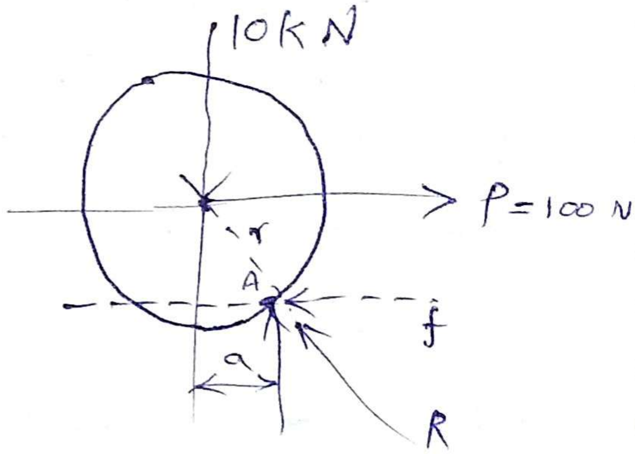
2. Determine horizontal force P required to move an automobile of mass 3000 kg along a horizontal road at constant speed. The diameter of each tyre is 1000 mm. Assuming coefficient of rolling resistance to be 2 mm and neglect all other forms of fric



$\Sigma M_A = 0$, $\quad -P \times 500 + (3000 \times 9.81) \times 2 = 0$

$\boxed{P = 117.72\ N}$

3. A sphere of 200 mm radius carries a load of 10 kN. If a horizontal force 100 N is necessary to move in a horizontal surface, determine the co-efficient of rolling resistance.



$\Sigma M_A = 0$,

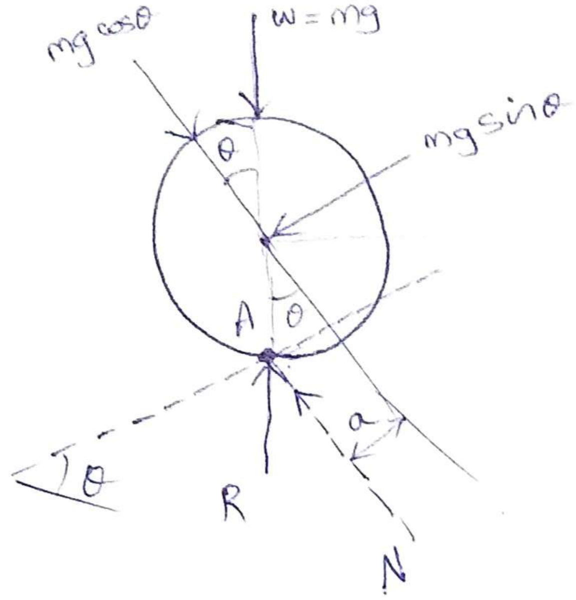
$(10 \times 10^3) \times a - (100 \times r) = 0$

$r = 200$ mm.

$\boxed{a = 2\ mm}$

5.1.9

4. A cylinder having a radius of 100 mm rolls down a slope 1 in 50. Determine the co-efficient of rolling resistance 'a' of the cylinder.



Given:-

The cylinder rolls down a slope 1 in 50.

It means, $\tan\theta = \frac{1}{50}$

$\Sigma M_A = 0,$

$$-(mg\cos\theta \times a) + (mg\sin\theta)(r) = 0$$

$$\cancel{mg}\cos\theta \times a = \cancel{mg}\sin\theta \times r$$

$$a = r\frac{\sin\theta}{\cos\theta}$$

$$= r\tan\theta$$

$$= 100\left(\frac{1}{50}\right)$$

$$\boxed{a = 2\text{ mm}}$$

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Dynamics of Particles

Part - 1 / Kinematics of Particle - (Translation)

Dynamics:

Dynamics is the branch of mechanics, which deals with the motion of particles or bodies under the action of forces. Dynamics is divided into two parts.
i) Kinematics, ii) kinetics

Kinematics is the study of motion of bodies without reference to the force which cause the motion. It is the study of "geometry of motion". It is used to relate displacement, velocity, acceleration and time without reference to the forces causing the motion.

Kinetics is the study of bodies with reference to the force which cause the motion. It is the study of the relationship between the forces acting on a body, the mass of the body and the motion of the body.

Types of Motion:

Motion:

A body or a particle is said to be in motion, if it changes the position with respect to a reference point.

Translation (or) Rectilinear Motion:

A type of motion is defined by the path traversed by it. (If the path is straight line, the motion is called the rectilinear motion or Translation.)

Curvilinear Motion:

If the path is a curved line, the motion is called curvilinear motion.

Pure rotation:

If the path is a circle, then the motion is called pure rotation.

* General Plane Motion:

If a body having both translation and rotation is said to be in general ~~pat~~ plane motion.

Rectilinear Motion:

Displacement:

The shortest distance bet. the initial & final position is called displacement.

Speed:

Speed rate of change of distance irrespective of the direction of motion of the body. Thus the speed may be defined of the magnitude of the velocity.

Velocity:

The velocity is defined ~~by~~ as the rate of change of displace

$$v = \frac{ds}{dt} \quad m/s.$$

Acceleration:

Acceleration is the rate of change of velocity and it is measured in m/s^2. The +ve acceleration is simply called as acceleration. Here the velocity increases w.r.to time. Negative acceleration is called deceleration where the velocity decreases with respect to time.

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Equation of motion for rectilinear motion - Constant acceleration ① Horizontal motion.

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

Where, $u \rightarrow$ be the initial velocity, m/s

$v \rightarrow$ be the final velocity, m/s

$t \rightarrow$ time, sec

$a \rightarrow$ acceleration, m/s^2

$s \rightarrow$ displacement, m.

② Vertical Motion: Motion under gravitational force

a) The downward motion equations are:-

$$v = u + gt$$

$$h = ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gh$$

b) the upward motion equations are:-

$$v = u - gt$$

$$h = ut - \frac{1}{2}gt^2$$

$$v^2 = u^2 - 2gh$$

Where, $g = 9.81\ m/s^2$ for downward motion

$g = -9.81\ m/s^2$ for upward motion

Basic equations:

$v = ds/dt$; $a = dv/dt$; $v\,dv = a\,ds$ [should be used for variable acceleration]

Numerical Problems:

1. A motion of a particle is described by an equation, displacement $s = 5t^2 - 7t + 2$. Find,

a) displacement, velocity, acceleration, when 't' = 2 sec

b) Minimum displacement and corresponding velocity and acceleration. Take 's' in 'm'.

soln:-

Displacement $S = 5t^2 - 7t + 2$

Differentiating w.r. to, t, $v = ds/dt = 10t - 7$

Differentiating w.r. to t, $a = \frac{dv}{dt} = 10$.

a) Displacement when $t = 2$ sec $\rightarrow$ $S_{t=2} = 5(2)^2 - 7(2) + 2$

$= 8\,m$

The velocity when $t = 2$ sec, $v_{t=2} = 10(2) - 7$

$= 13\,m/s$.

Acceleration is constant, $a = 10\,m/s^2$ all the time

b) For minimum displacement, the first derivative

$\frac{ds}{dt}$ i.e.) $v = 0$

$10t - 7 = 0$

$t = \frac{7}{10}$ sec.

$\therefore$ corresponding displacement, $S = 5\left(\frac{7}{10}\right)^2 - 7\left(\frac{7}{10}\right) + 2$

$= -0.45\,m$

Acceleration $a = 10\,m/s^2$ always.

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2. A motion of a particle is defined by $S = 2t^3 - 6t^2 + 15$ where S in meter and t is in sec. Determine position and acceleration when.
i) velocity is zero ii) Velocity is minimum

Soln:-

$$S = 2t^3 - 6t^2 + 15$$

$$v = \frac{ds}{dt} = 6t^2 - 12t$$

$$a = \frac{dv}{dt} = 12t - 12,$$

i) When velocity is zero,

$$v = [6t^2 - 12t] = 0$$

$$6t(t-2) = 0$$

$t \neq 0$ or $t = 2$ sec.

∴ Displacement, $S = 2(2^3) - 6(2^2) + 15 = 7$ m.
Acceleration, $a = 12(2) - 12 = 12\ m/s^2$.
$\cancel{v = 6(2^2) - 12(2) =}$

ii) When the velocity is minimum, the first derivative

$$\frac{dv}{dt} = a = \text{zero}.$$

$$12t - 12 = 0$$

$$t = 1 \text{ sec.}$$

∴ Displacement $S = 2(1)^3 - 6(1)^2 + 15 = 11$ m.

Velocity $v = 6(1)^2 - 12(1)$

$= -6$ m/s

4. A car moves in a straight line for a short time, its velocity is defined as $V = 3t^2 + 2t$ m/s. Determine its position and acceleration at $t = 3$ sec. Also assume that initial displacement is negligible.

Soln:- Given $V = 3t^2 + 2t$; $a = 6t + 2$ —— ①

Differentiate w.r.t time, then acceleration, $a = 6t + 2$.

take $\frac{ds}{dt} = V = 3t^2 + 2t$

$$ds = (3t^2 + 2t)\,dt$$

Integrating, $S = t^3 + t^2 + C$

When, $t = 0$, $S = 0$ [Given]

$$\therefore C = 0$$

$$S = t^3 + t^2 \quad \text{—— ②}$$

① $\Rightarrow$ $\therefore$ Acceleration when $t = 3$ sec, $a_{t=3} = 6(3) + 2$

$$= 20\ m/s^2.$$

② $\Rightarrow$ Displacement at $t = 3$ sec, $S_{t=3} = (3)^3 + (3)^2$

$$= 36\ m.$$

---

5. The velocity of a particle along x axis is gn by $V = 5s^{3/2}$ where s is in 'm', v is in m/s, Determine the acceleration, when $s = 2$ m.

Given:- $V = 5s^{3/2}$

$$\frac{dv}{dt} = a = \frac{3}{2}(5)\, s^{1/2} \cdot \frac{ds}{dt}$$

$$a = 5\left(\frac{3}{2}\right)s^{1/2} \cdot V = 5\left(\frac{3}{2}\right)s^{1/2} \times 5s^{3/2} = \frac{75}{2}$$

When, $s = 2$ m; $a = \frac{75}{2}(2^2)$

$$\boxed{a = 150\ m/s^2} \text{ at } s = 2\,m$$

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8. The speed of a particle is given by $v = 2t^3 + 5t^2$. What distance does it travel while its speed increases from 7 m/s to 99 m/s?

Given: $v = 2t^3 + 5t^2$ ——— (1)

$$v = \frac{ds}{dt} = 2t^3 + 5t^2$$

$$ds = (2t^3 + 5t^2)\,dt$$

$$\int ds = \int (2t^3 + 5t^2)\,dt$$

$$S = \int_{t \text{ when } v = 7\,m/s}^{t \text{ when } v = 99\,m/s} (2t^3 + 5t^2)\,dt \quad \text{——— (2)}$$

Finding ~~limits~~ out the time limits:

When $v = 7$ m/s

eqn (1) becomes, $7 = 2t^3 + 5t^2$

$$2t^3 + 5t^2 - 7 = 0 \qquad [at^3 + bt^2 + ct + d = 0]$$

$$t = 1 \text{ sec.}$$

When, $v = 99$ m/s

eqn (1) becomes, $99 = 2t^3 + 5t^2$

$$2t^3 + 5t^2 - 99 = 0$$

$$t = 3 \text{ sec.}$$

eqn (2) becomes, $S = \int_1^3 (2t^3 + 5t^2)\,dt = \left[\frac{2t^4}{4} + \frac{5t^3}{3}\right]_1^3$

$$= \left[\frac{3^4}{2} + \frac{5(3)^3}{3}\right] - \left[\frac{1}{2} + \frac{5}{3}\right] = 83.333 \text{ m.}$$

9. A particle starting from rest, moves in a straight [line]
and its acceleration is given by $a = 50 - 36t^2 \ m/s^2$
Determine the velocity of the particle when it has
travelled 52 m.

Soln:- Given $a = 50 - 36t^2$ ——— ①

$$\frac{dv}{dt} = 50 - 36t^2$$

$$dv = (50 - 36t^2)\,dt$$

Integrating, $v = 50t - (36)\frac{t^3}{3} + C_1$

when, $t = 0$, $v = 0$ [Initial position] conds,

so; $C_1 = 0$

$$v = 50t - 12t^3 \text{ ——— ② } = \frac{ds}{dt}$$

and, $\frac{ds}{dt} = 50t - 12t^3$.

$$ds = (50t - 12t^3)\,dt$$

Integrating, $S = \frac{50t^2}{2} - 12\frac{t^4}{4} + C_2$

$$= 25t^2 - 3t^4 + C_2$$

when $t = 0$, $s = 0$ $\therefore$ $C_2 = 0$.

$$S = 25t^2 - 3t^4 \text{ ——— ③}$$

Given, $S = 52$ m . . . finding out 't'.

eqn ③ becomes, $52 = 25t^2 - 3t^4$ ——— ④

Put $t^2 = x$ in eqn ④

$$52 = 25x - 3x^2 \qquad Ax^2 + Bx + C = 0$$

solving, $x = 4,\ 4.3333$

$$t = \pm\sqrt{4} \text{ and } \pm\sqrt{4.3333}$$

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$t = 2.0816$ and 2 sec.

When, $t = 2$ sec . . . . $v = 50(2) - 12(2)^3$

$= 4$ m/s.

When, $t = 2.0816$ sec . . . . $v = 50(2.0816) - 12(2.0816)^2$

$= -4.163$ m/s.

---

10. An automobile travels 360 m in 30 s while being accelerated at a constant rate of $0.5\ m/s^2$. Determine,
a) Initial velocity b) Final velocity c) distance travelled during the first 10 sec.

Soln :-

1. Initial velocity :-

360 m
$t = 30s$
$a = 0.5\ m/s^2$
$S_{t=10} = ?$

Displacement $S = ut + \frac{1}{2}at^2$

$360 = u(30) + \frac{1}{2}(0.5)(30)^2$

$\boxed{u = 4.5\ m/s}$

2. Final velocity :

$v = u + at$

$= 4.5 + (0.5)(30)$

$\boxed{v = 19.5\ m/s}$

3. $S_{t=10} = ?$

$S = ut + \frac{1}{2}at^2$

$= 4.5(10) + \frac{1}{2}(0.5)(10^2)$

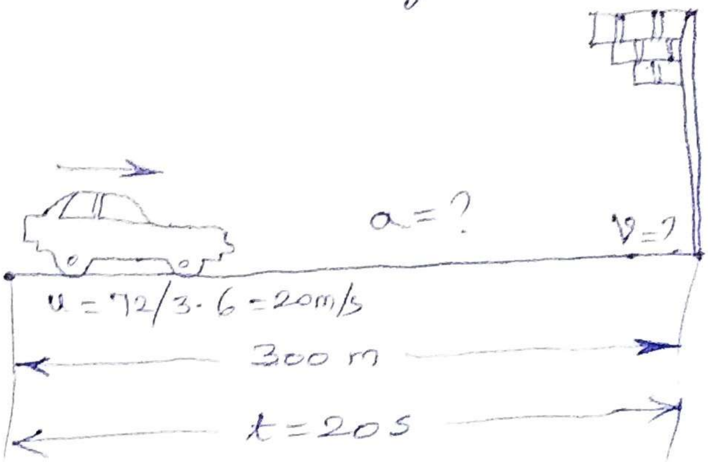
$\boxed{S_{t=10} = 70\ m}$

---

12. A driver of a car travelling at 72 km/h observes the traffic light 300 m ahead of him turning red. The traffic light is timed to remain red for 20 s before it turns green. If the motorist wishes to pass the light without stopping to wait for it to turn green, determine, (i) the required uniform acceleration of the car: (ii) the speed with which the motorist crosses the traffic light.

$u = 72\ km/h$

$u = \frac{72 \times 1000}{60 \times 60} = 20\ m/s$



Displacement $S = ut + \frac{1}{2}at^2$

$300 = 20 \times 20 + \frac{1}{2}(a)(20)^2$

$a = -0.5\ m/s^2$ (Deceleration).

Final velocity, $V = u + at$

$= 20 + (-0.5 \times 20)$

$V = 10\ m/s$

$= \frac{10 \times 60 \times 60}{1000} = \frac{36000}{1000}$

$V = 36\ km/h$

Answers:

i) uniform deceleration is $0.5\ m/s^2$

ii) Speed at which motorist cross the traffic light $\Big\} = 36$ kmph.

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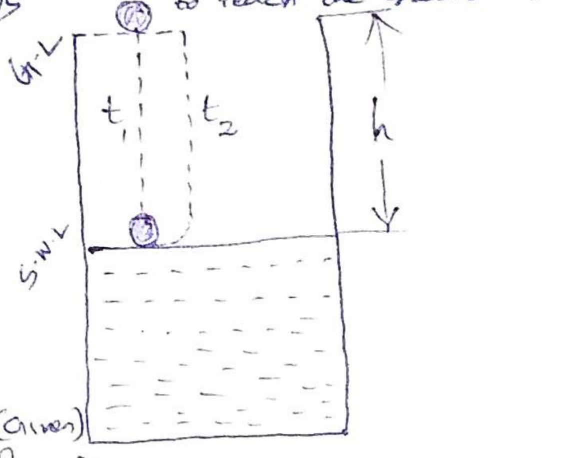
13. A Stone is dropped into a well. The sound of the splash is heard 3.63 seconds later. How far below the ground is the surface of the water? Assume the velocity of sound as 331 m/s.

Let $t_1 \to$ Time req. to reach the stone to surface of water

$t_2 \to$ Time taken by sound to reach the ground Level

$t_1 + t_2 = 3.63$ sec ; $u = 331$ m/s



Consider Stone:

u = Initial velocity $= 0$

$a = +g$

Displacement $= h$

$$h = \cancel{ut_1} + \frac{1}{2} g t_1^2 \quad \text{——— (1)}$$

Consider Sound:

Uniform velocity of sound, $u = 331$ m/s (Given)

$a = 0$ or $g = 0$. ($\because$ uniform velocity)

$$\text{So, } h = u t_2 + \cancel{\frac{1}{2} g t_2^2} \quad \text{——— (2)}$$

$$h = 331 \times t_2 \quad \text{——— (2)}$$

$$t_1 + t_2 = 3.63 \Rightarrow t_2 = 3.63 - t_1 \quad \text{——— (3)}$$

Equating (1) & (2)

$$\frac{1}{2} g t_1^2 = 331 \times t_2$$

$$\frac{1}{2}(9.81) t_1^2 = 331 \times (3.63 - t_1) \quad [\because \text{eqn } (3)]$$

$$4.905 t_1^2 + 331 t_1 - 1201.53 = 0$$

$$\boxed{t_1 = 3.45} \text{ Sec. or } -70.9$$

$$\therefore h = \frac{1}{2} g t_1^2 \quad \text{——— (1)}$$

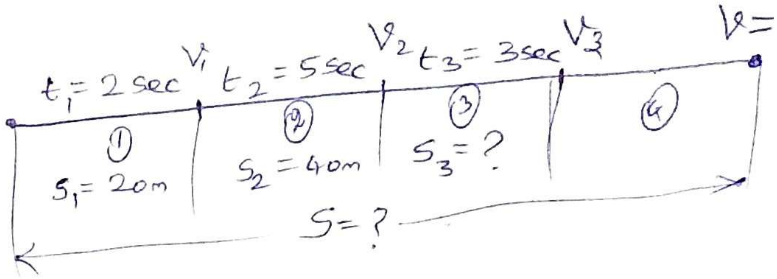
$$h = \frac{1}{2}(9.81) \times 3.45^2$$

$$h = 58.4935 \text{ m}$$

The surface of water is 58.4935 m below the ground.

15. A particle under constant deceleration is moving in a straight line and covers a distance of 20 m in the first 2 seconds and 40 m in the next 5 sec. Calculate the distance it covers in the subsequent 3 sec and total distance travelled by the particle before it comes to rest.

Let the initial velocity is 'u' and be the deceleration is 'a'.



Phase 1:

Here, $t = 2$ sec, $S = 20$m

$$S = ut + \frac{1}{2}at^2$$

$$20 = u(2) + \frac{1}{2}(a)(2^2)$$

$$20 = 2u + 2a$$

$$u + a = 10 \quad \text{——— (1)}$$

Phase 1 & 2 :-

Here, $t = 7$ sec, $S = 60$m

$$60 = u(7) + \frac{1}{2}(a)(7^2)$$

$$60 = 7u + 24.5a$$

$$7u + 24.5a = 60 \quad \text{——— (2)}$$

solving 1 & 2

$$u = 10.57142 \text{ m/s}, \quad a = -0.5714 \text{ m/s}^2$$

$S = ut + \frac{1}{2}at^2$

Consider Phase ③ :- here, $t = 10$ sec, $u = 10.5714$ m ; $a = -0.5714$ m/s²

- Phase ③ distance = up to phase ③ distance — up to phase ② distance

$$S_3 = \left\{10.57142 \times 10 + \frac{1}{2}(-0.571428)(10^2)\right\} - (60)$$

$$S_3 = 17.1428 m$$

$$V = u + at$$

$$V_3 = u + at$$

$$= 10.57142 + (-0.571428)(10)$$

$$V_3 = 4.85714 \text{ m/s}$$

Consider Phase ④:

Here, $u_4 = 4.85714$ m/s, [here final velocity of V_3 is the initial velocity of phase ④

because $V_4 = 0$

$$a = -0.571428 \text{ m/s}^2$$

$$V_4 = u_4 + at_4$$

$$0 = 4.85714 + (-0.571428)(t_4)$$

$$t_4 = 8.5 \text{ sec.}$$

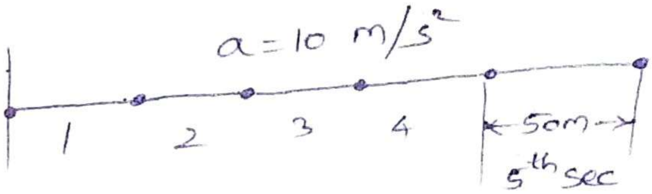
The total time $= 2 + 5 + 3 + 8.5 = 18.5$ Sec.

$S = ut + \frac{1}{2}at^2$

Total distance travelled, $S = 10.57142 \times 18.5 + \frac{1}{2}(-0.571428) \times 18.5^2$

$$\boxed{S = 97.785 \text{ m}}$$

16. A particle moving with an acceleration $10 m/s^2$ travels a distance of 50 m, during 5^{th} sec. Find its Initial Speed.



Distance travelled in the n^{th} sec is gn by

$$\boxed{S_n^{th} = u + \frac{a}{2}(2n-1)}$$

$$S_5^{th} = u + \frac{10}{2}(2(5)-1)$$

$$50 = u + 45$$

$$\boxed{u = 5 m/s}$$

---

17. A body is moving with uniform acceleration and covers the 20 m in 4^{th} sec and 30 m in 8^{th} sec. Determine (i) the initial velocity of the body ii) acceleration of the body iii) distance travelled during 10^{th}

Soln:-

Distance covered in n^{th} sec, $S_n^{th} = u + \frac{a}{2}(2n-1)$

Apply the given conditions,

$$20 = u + \frac{a}{2}(8-1)$$

$$20 = u + 7(a/2) \quad \text{---} ①$$

$$30 = u + 15(a/2) \quad \text{---} ②$$

$$② - ① \Rightarrow 10 = 8(a/2)$$

$\boxed{a = 2.5 m/s^2}$, Substitute in ①

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$$20 = u + \left(7 \times \frac{2.5}{2}\right)$$

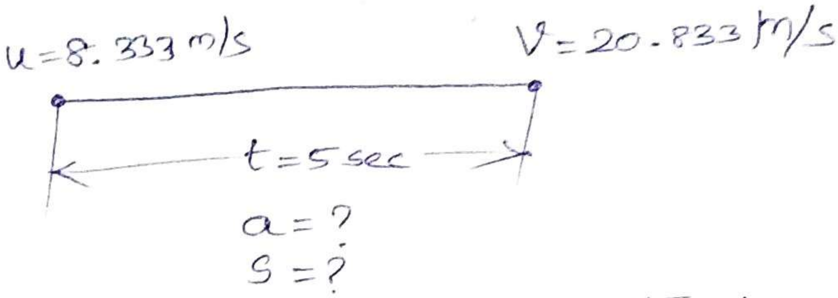
$$\boxed{u = 11.25\ m/s}$$

Distance travelled during 10^{th} sec,

$$S_{10}^{th} = 11.25 + \frac{2.5}{2}\left[2(10) - 1\right]$$

$$\boxed{S_{10}^{th} = 35\ m}$$

18\. A car accelerated uniformly from a speed of 30 kmph to a speed of 75 km/h in 5 sec. Determine the acceleration of the car and also the distance travelled during 5 sec.



$$u = \frac{30 \times 1000}{60 \times 60} = 8.333\ m/s\ ; \quad V = \frac{75 \times 1000}{60 \times 60} = 20.833\ m/s$$

$$V = u + at$$

$$20.833 = 8.333 + a(5)$$

$$\boxed{a = 2.5\ m/s^2}$$

$$S = ut + \frac{1}{2}at^2$$

$$= 8.333(5) + \frac{1}{2}(2.5)(5^2)$$

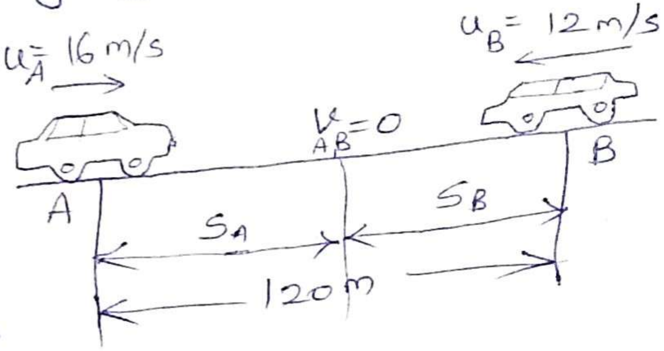
$$\boxed{S = 72.915\ m}$$

18. Two cars are travelling towards each other on a lane road at 16 m/s and 12 m/s resp. When 120 m apart, both drivers realize to apply brakes. The succeed in stopping simultaneously and just short of collision. Assuming constant deceleration of each car. Determine,

a) time required for the cars to stop

b) deceleration of each car

c) distance travelled by each car.



Consider car 'A' :-

$S_A = 16t + \frac{1}{2} a_A t^2$ —— (1)

$v_A = 16 + a_A t = 0$

$a_A = \frac{-16}{t}$, Put in (1)

$S_A = 16t - \frac{16}{2t} t^2$; $S_A = 8t$

Consider car 'B' :-

$S_B = 12t + \frac{1}{2} a_B t^2$ —— (2)

$v_B = 12 + a_B t = 0$; $a_B = \frac{-12}{t}$, subs in (2)

$S_B = 12t - \frac{12}{2t} t^2$

$S_B = 6t$

Total distance = Displacement by A + Displacement by B

$120 = S_A + S_B = 8t + 6t$

$t = \frac{120}{14} = 8.571$ sec.

$a_A = \frac{-16}{t} \Rightarrow a_A = -1.8667$ m/s² ; $a_B = -1.4$ m/s. $[\because a_B = \frac{-12}{t}]$

$S_A = 68.571$ m ; $S_B = 51.429$ m

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22. A ball is thrown vertically up. It was found to travel a distance of 5m. during its 3rd second. Find the initial velocity.

$$S_n^{th} = u + \frac{a}{2}(2n-1)$$

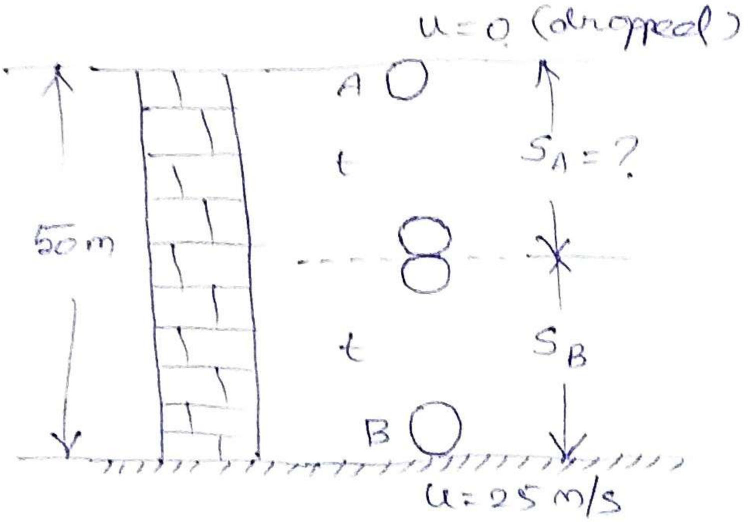
Here, $a = -g$; $S_n^{th} = 5\,m$; $n = 3$

$$5 = u + \left(\frac{-9.81}{2}\right)(2\times3-1)$$

$$\boxed{u = 29.525\ m/s}$$

---

23. A stone is dropped from the top of a tower 50m high. At the same time another stone is thrown up from the foot of the tower with a velocity of 25 m/s. At what distance from the top and how much time after, the two stones meet each other.



Consider stone 'A':

$u = 0, \; a = g, \; \text{time} = t$

$\therefore S_A = 0(t) + \frac{1}{2} g t^2$ ——— (1)

Consider stone 'B':

$u = 25 \text{ m/s}, \; a = -g, \; \text{time} = t$

$S_B = 25t - \frac{1}{2} g t^2$ ——— (2)

From schematic sketch, $S_A + S_B = 50$

$$\cancel{\left[\frac{1}{2} g t^2\right]} + \left[25t - \cancel{\frac{1}{2} g t^2}\right] = 50$$

$$25t = 50$$

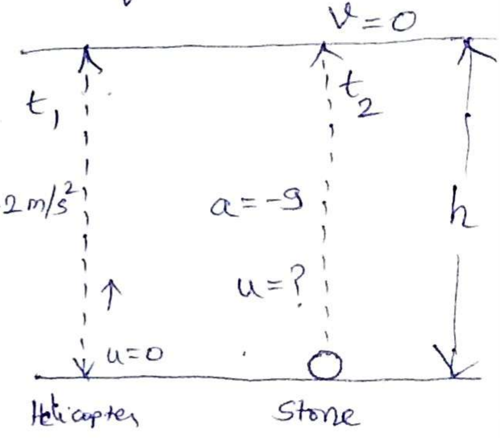
$$\boxed{t = 2 \text{ sec}}$$

$S_A = \cancel{6}\ \frac{1}{2}(9.81) \times 2^2$ (From (1))

$$\boxed{S_A = 19.62 \text{ m}}$$

---

26. A helicopter raises from ground with a constant acceleration of 1.2 m/s^2. After 4 seconds, a stone is thrown vertically up from the launching pad. What is the initial velocity of stone if the stone just touches the helicopter?



Soln:-

h → distance covered by both helicopter and stone.

t_1 → time taken by helicopter

t_2 → time taken by stone

$t_1 = t_2 + 4$ ——— (1)

Find "h" for helicopter :

$$h = ut_1 + \frac{1}{2} g t_1^2$$

$$h = \frac{1}{2} g_{(heli)} t_1^2 \qquad [\because u = 0] \text{ ——— } (2)$$

Find "h" for stone :-

$$h = ut_2 - \frac{1}{2} g_{(s)} t_2^2$$

also $v = u - g t_2$

$$0 = u - g t_2$$

$$u = g_{(s)} t_2 \text{ ——— } (4)$$

so, $h = g_{(s)} t_2^2 - \frac{1}{2} g_{(s)} t_2^2$

$$h = \frac{1}{2} g_{(s)} t_2^2 \text{ ——— } (3)$$

w.k.t.

(2) = (3)

$$\frac{1}{2} g_{(heli)} t_1^2 = \frac{1}{2} g_{(s)} t_2^2$$

$$t_1^2 = \frac{g_{(s)}}{g_{(heli)}} t_2^2$$

$$t_1 = \sqrt{\frac{g_{(s)}}{g_{(heli)}}} \, t_2$$

$$t_1 = \sqrt{\frac{9.81}{1.2}} \, t_2$$

$$t_1 = 2.8591 \, t_2$$

so, $2.8591 t_2 = t_2 + 4$ [eqn (1)]

$$t_2 = 2.15146 \text{ sec}$$

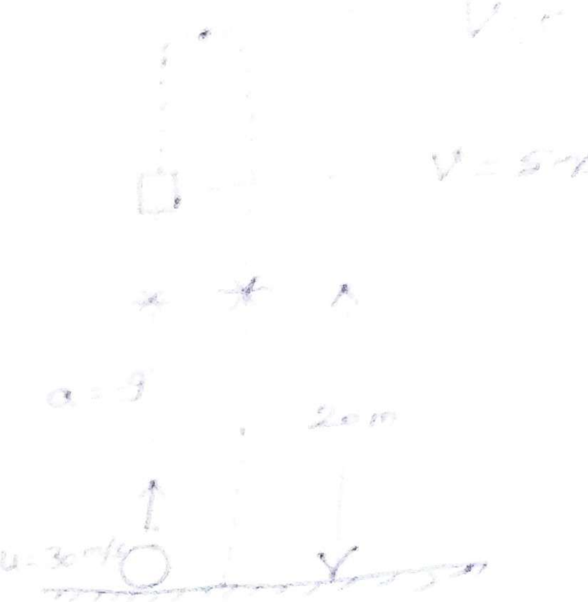
from (4) $\Rightarrow$ $u = 9.81 \times 2.15146$

$$\boxed{u_{(s)} = 21.1 \text{ m/s}}$$

27. A ball is thrown vertically up with a velocity of 30 m/s. Determine the time (i) when the ball is 20 m above the point of projection (ii) when the velocity will be 5 m/s. iii) when it returns back to original position.

Soln

i) $u = 30$ m/s, $a = -g$, $h = 20$ m



$h = ut - \frac{1}{2} g t^2$

$20 = 30t - \frac{1}{2}(9.81)(t^2)$

$4.905t^2 - 30t + 20 = 0$

$t = 0.7614$ sec while going up

$t = 5.3526$ sec while coming down

ii) $u = 30$ m/s, $a = -g$, $V = 5$ m/s

$V = u - gt$

$5 = 30 - 9.81t$

$t = 2.545$ sec

iii) Time required to reach maximum height is t.

The Total time required to return its original position is gn by, $t_T = 2t$

$u = 30$ m/s, $V = 0$, $a = -g$.

$V = u - gt$

$0 = 30 - g(t)$

$t = 3.0581$ sec

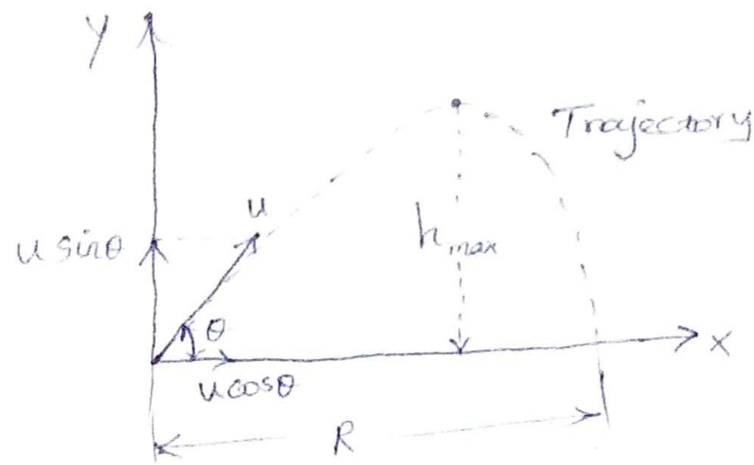
Total time $= 6.1162$ sec.

Kinematics of Particle (curvilinear Motion)

Curvilinear Motion

If the path traversed by a particle is a curv[e] then the motion is called curvilinear motion. It is having both x and y direction displacements.

Eg. Projectile motion ie has combined effect of a vertical and a horizontal motion.



Projectile motion:

1. Velocity of projection (u):-

The velocity with a particle of projected is called as velocity of projection.

2. Angle of projection (θ):-

The angle between the direction of projection and the horizontal direction is called as angle of Projec[tion]

3. Trajectory:

The path traced by the projectile is called as its trajectory.

The x & y component of velocity

$$V_x = 8 \cos 63.43 = 3.75 \text{ m/s}$$

$$V_y = 8 \sin 63.43 = 7.1554 \text{ m/s}$$

Radius of curvature $\rho = \dfrac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{d^2y/dx^2}$

$$= \frac{(1+2^2)^{3/2}}{2/3}$$

$$= 16.7705 \text{ m}$$

$$a_n = \frac{V^2}{\rho} = \frac{64}{16.7795} = 3.8162 \text{ m/s}^2$$

$a_t = 0$ (const. speed)

$a = 3.8162$ m/s.

---

3. The motion of a particle is described by $x = t^2 + 8t + 4$ and $y = t^3 + 3t^2 + 8t + 4$. Find a) Initial velocity of the Particle b) Velocity of the particle at $t = 2$ sec. c) acceleration at $t = 2$ sec

Soln:-

$x = t^2 + 8t + 4$

$V_x = \dfrac{dx}{dt} = 2t + 8$

$a_x = \dfrac{d^2x}{dt^2} = 2$

$y = t^3 + 3t^2 + 8t + 4$

$V_y = \dfrac{dy}{dt} = 3t^2 + 6t + 8$

$a_y = \dfrac{d^2y}{dt^2} = 6t + 6$

a) Initial Velocity

Initially $t=0$; $V_x = 8\ m/s$

$V_y = 8\ m/s$

$V = \sqrt{8^2+8^2}$

$= 11.3137\ m/s$ at $\theta_1 = 45^\circ$

b) Velocity at $t = 2$ sec.

$V_x = 12\ m/s$

$V_y = 32\ m/s$

$V = \sqrt{12^2+32^2}$

$= 34.176\ m/s$ at $\theta_2 = 69.443^\circ$

c) Acceleration at $t=2$ sec

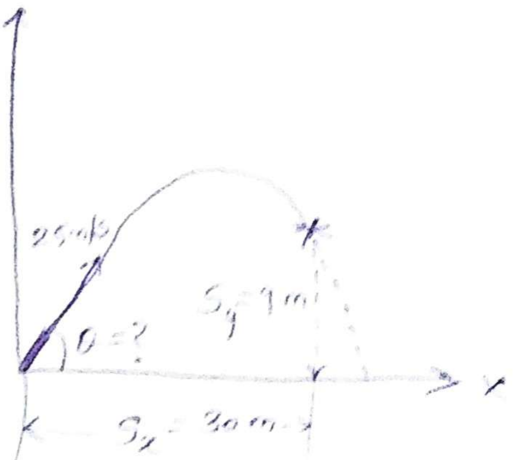
$a_x = 2\ m/s$

$a_y = 18\ m/s$

$a = \sqrt{2^2+18^2}$

$= 18.11\ m/s$ at $\theta_3 = 83.6598^\circ$

5. A bird is sitting on the top of a tree 9m high. At what angle of elevation should a person standing at a distance of 30 m from the tree, project a stone with a velocity of 25 m/s, so as to hit the bird as soon as possible?



x direction	y direction
$U_x = 25\cos\theta$	$u_y = 25\sin\theta$
$a_x = 0$	$a_y = -g$
$S_x = 30m$	$S_y = 9m$
$S_x = u_x t + \frac{1}{2}a_x t^2$	$S_y = u_y t + \frac{1}{2}a_y t^2$
$30 = 25\cos\theta \times t$ — ①	$9 = 25\sin\theta(t) - \frac{1}{2}(g)t^2$ —— ②

① ⇒ $t = \dfrac{30}{25\cos\theta}$

Subs' t' value in ②

$$9 = \left(25\sin\theta \times \frac{30}{25\cos\theta}\right) - \left(\frac{1}{2}(9.81) \times \frac{30^2}{25^2\cos^2\theta}\right)$$

$$9 = 30\tan\theta - 7.0632\sec^2\theta \qquad \left[\sec^2\theta = \frac{1}{\cos^2\theta}\right]$$

$$= 30\tan\theta - 7.0632(1+\tan^2\theta)$$

$$9 = 30\tan\theta - 7.0632 - 7.0632\tan^2\theta$$

$$7.0632\tan^2\theta - 30\tan\theta + 16.0632 = 0$$

$$\tan\theta = 3.61894,\ 0.6284$$

$$\theta = 74.553^\circ,\ 32.1468^\circ$$

Time of flight is given by,

$t = \frac{30}{25 \cos\theta}$ [∵ eqn (1)]

If $\theta = 74.553$, $t = 4.5054$ sec

$\theta = 32.1468^\circ$, $t = 1.4173$ sec

$\boxed{\theta = 32.1468^\circ}$ because it takes less time as compared with that of the other angle of projectio

---

16. A particle is projected in air with a velocity 100 m/s and at an angle of 30° with the horizontal, find (i) the horizontal range ii) the max hight reached by the particle iii) the time of flight.

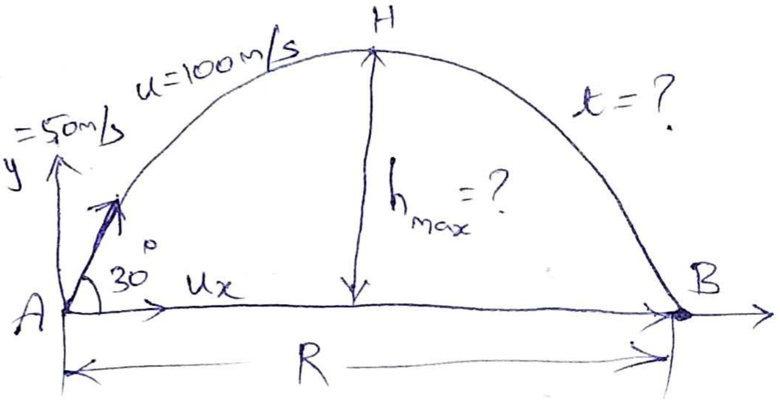
x-direction

$u_x = 100 \cos 30 = 86.6$ m/s

$a_x = 0 = g_x = 0$

$S_x = R = u_x t + \frac{1}{2} g_x t^2$

$S_x = R = u_x t$ — (1)



y direction:

$u_y = 100 \sin 30 = 50$ m/s

$a_y = -9.81$ m/s^2

At point B, Vertical displacement $S_y = 0$

$S_y = u_y t + \frac{1}{2} a_y t^2$

$0 = 50 \times t - \frac{1}{2}(9.81) t^2 \Rightarrow t = 10.193$ sec.

(1) $\Rightarrow S_x = R = u_x \times t \Rightarrow R = 882.72$ m.

At max. height H, Vertical velocity is zero.

$\boxed{h_{max} = 127.4}$

$\cancel{h_{max} = \frac{u^2 \sin^2\theta}{2g}}$ $v_y^2 = u_y^2 + 2 a_y h_{max} \Rightarrow 0 = 50^2 - (2 \times 9.81) \times h_{max}$

Kinematics of Particle - Relative Motion

Relative Velocity:

The motion of a body with respect to another moving body is known as relative motion.

The relative position of B with respect to A and denoted by,

$$S_{B/A} = S_B - S_A$$

$$S_B = S_A + S_{B/A}$$

The relative velocity of B with respect to A and is denoted by $v_{B/A} = v_B - v_A$

$$v_B = v_A + v_{B/A}$$

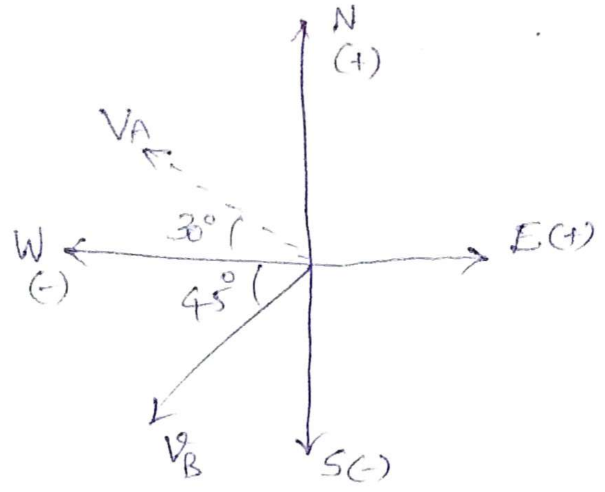
The relative acceleration of B w.r. to A and is defined as.

$$a_{B/A} = a_B - a_A$$

$$a_B = a_A + a_{B/A}$$

---

1. Two ships moves from a point at the same time. Ship A has a velocity of 30kmph and is moving in NW 30° while ship B is moving in South-west direction with a velocity of 40 kmph. Determine the relative velocity of A with r. to B and distance between them after half an hour.



$V_A = 30\,kmph$

$V_B = 40\,kmph$

$V_{Ax} = -30 \cancel{\sin} \cos 30 = -25.98$ $\qquad$ $V_{Bx} = -40 \cos 45 = -28.28$

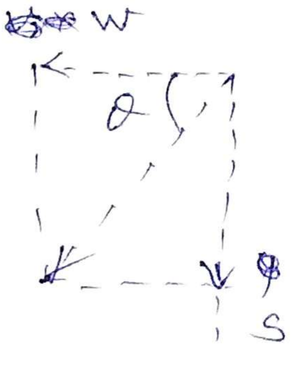
$V_{Ay} = +30 \sin 30 = 15$ $\qquad$ $V_{By} = -40 \sin 45 = -28.28$

$V_{(A/B)x} = V_{Ax} - V_{Bx} = -54.26\ kmph$

$V_{(A/B)y} = V_{Ay} - V_{By} = -13.28\ kmph.$

$V_{A/B} = \sqrt{54.26^2 + 13.28^2} = 55.86\ kmph$

$\tan\theta = \frac{13.28}{54.26}$



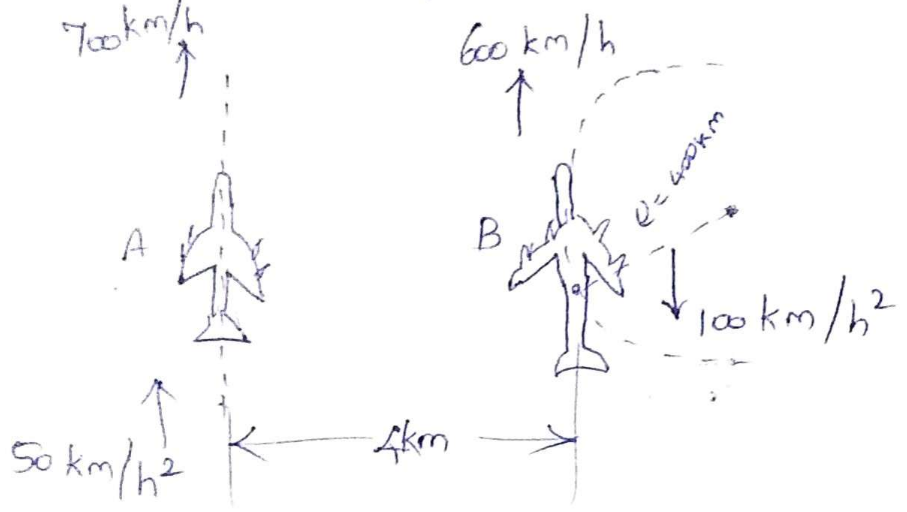
$\theta = 13.75^\circ$

Relative distance after half an hour $\Big\} = V_{A/B} \times time$

$= 55.86 \times \frac{1}{2}$

$= 27.933\ km$

3. Plane A is flying along a straight line path, where as plane B is flying along circular path having a radius of curvature of ρ_B is 400 km shown in fig. Determine the velocity and acceleration of B as measured by the pilot of A.



Soln:-

* Assume the upward and the right side quantity as +ve.

* The plane A translates where as plane B has curvilinear motion.

1. Velocity :-

$$V_{B/A} = V_B - V_A$$

$$= 600 - 700 = -100 \text{ kmph} = 100 \text{ kmph} (\downarrow)$$

2. Acceleration :-

$$a_{Ax} = 0 \quad ; \quad a_{Ay} = 50 \text{ km/h}^2$$

$$a_{Bx} = \frac{V_B^2}{\rho} = \frac{600^2}{400} = 900 \text{ km/h}^2 \quad ; \quad a_{By} = -100 \text{ km/h}^2.$$

$$a_{(B/A)x} = a_{Bx} - a_{Ax} = 900 \text{ km/h}^2 \quad ; \quad a_{(B/A)y} = a_{By} - a_{Ay} = -100 - 50 = -150$$

$$a_{(B/A)} = 912.41 \text{ km/h}^2 \quad ; \quad \theta = \tan^{-1}\left(\frac{150}{900}\right) = 9.46^\circ.$$

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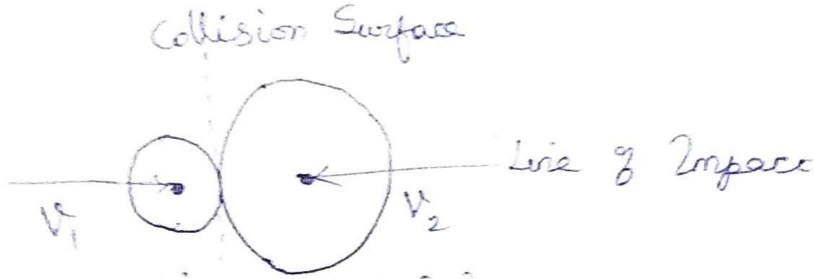
Impact of Elastic Bodies

A collision between two bodies to be an impact, if the bodies are in contact for a short interval of a time and exert very large force on a shot period of time. On impact, the bodies deform first and then recover due to elastic properties and start moving with different velocities.



Direct Central Impact

Definitions:

Line of Impact:

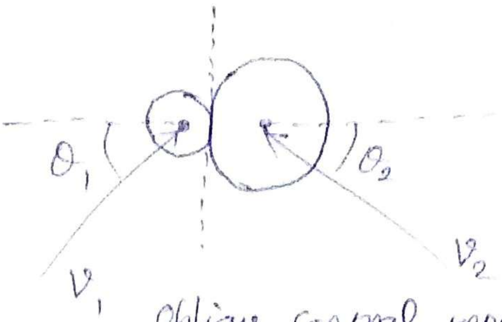
The line drawn perpendicular to colliding surface is called 'line of impact'.

Direct Impact:

If the direction of velocity of both colliding bodies are directed along the line of impact, then the collision is called the direct impact.

Oblique Impact:

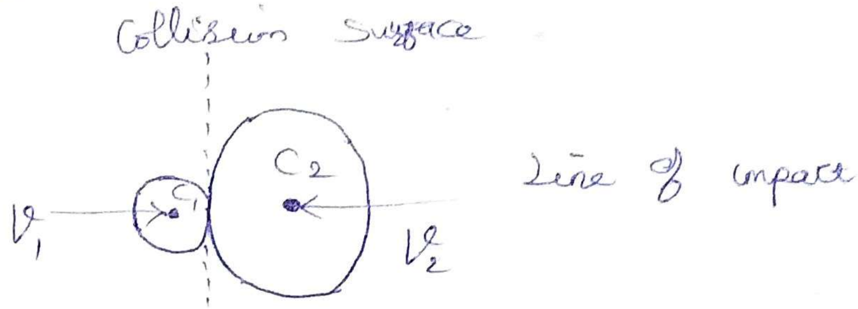
If direction of motion of one or both the bodies are not directed along the line of impact, then the collision is called Oblique Impact.



Oblique central impact

Central Impact:

If the mass centre of colliding bodies are on the line of Impact then the impact is called central impact.



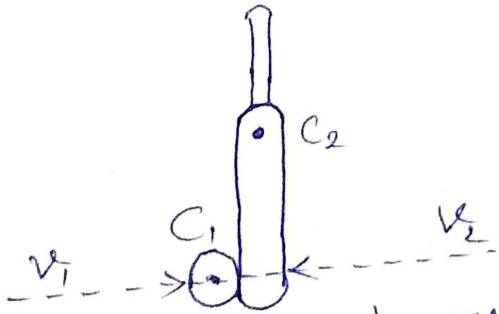
Central Impact

Eccentric Impact:

Even if the mass centre of one of the colliding bodies are not on the line of impact, then the impact is called Eccentric impact.

Eg. Cricket bat & ball.

C_1 and C_2 be the centre of ball and bat. They are not located on line of Impact.



Eccentric Impact

Period of Collision:

During the collision, the bodies undergo a deformation for a small time interval and then recover the deformation in a further small interval. Time elapse between initial contact and maximum deformation is called period of deformation.

And the time elapse between maximum deformation and the instant of seperation is called time of restitution or Period of recovery.

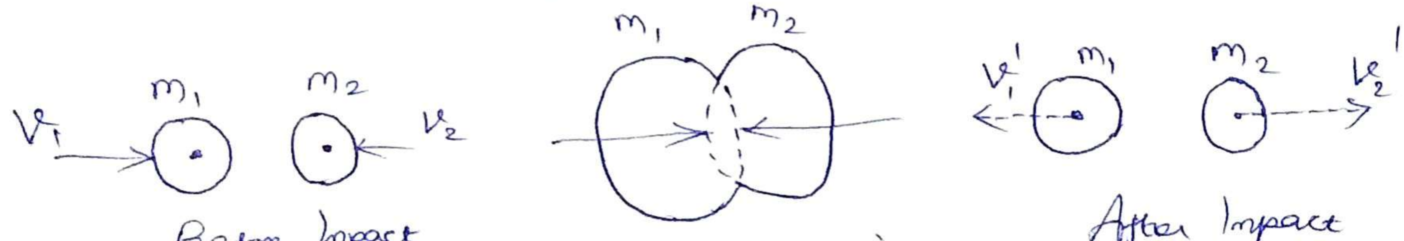
Perfectly elastic Impact [e = 1]

If both of bodies ~~do not return~~ ~~regain~~ regain to their original Shape and size after the collision. Both momentum and energy conserved.

Inelastic Impact [e > 1]

If both of bodies ~~regain~~ dont return to their original Shape and size completely, after the collision, Only the momentum remains conserved but there is a loss of energy.

Principle of Collision:



Before Impact — Maximum Deformation — After Impact

Consider 2 bodies approach each other with the velocity V_1 & V_2 and masses m_1 and m_2 as shown in f

Let F be the force exerted due to collision at a small time. Apply conservation of momentum principle for both bodies.

$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$$

The co-efficient of restitution is the ratio of the magnitude of impulse during the restitution period and deformation period.

Also it is the ratio of relative velocity of separa to velocity of approach.

$$\text{Co-efficient of restitution, } e = \frac{\text{Relative velocity of separa}}{\text{" of appro}}$$

$$= \frac{V_2' - V_1'}{V_1 - V_2}$$

This is called Newton's law of collision or restitut

The collision problem can be solved by applying,

1. Conservation of momentum and 2. Newton's law of Col

Important Cases:

1. When $e = 1$, $m_1 = m_2 = m$

We can apply both conservation of momentum and conservation of energy principles.

$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$$
$$mV_1 + mV_2 = mV_1' + mV_2' \quad [m_1 = m_2 = m]$$
$$V_1 + V_2 = V_1' + V_2' \text{ ——— } (1)$$

$$e = 1 = \frac{V_2' - V_1'}{V_1 - V_2} \text{ ——— } (2)$$

$$V_1 - V_2 = V_2' - V_1' \text{ ——— } (2)$$

$(1) + (2) \Rightarrow \quad 2V_1 = 2V_2'$

$$V_2' = V_1$$

$(1) - (2) \Rightarrow \quad V_1' = V_2$

(After an elastic impact of two equal masses exchange their velocities)

2. Perfectly Plastic bodies, $e = 0$

$$e = 0 = \frac{V_2' - V_1'}{V_1 - V_2}$$

$$V_2' - V_1' = 0$$

$$V_2' = V_1'.$$

After impact, the final velocity of both the bodies are equal, it means they move together as a single body. Collision is said to be perfectly inelastic, both the particle stick together after collision and move with the same velocity.

3. ~~If~~ $m_2 >>> m_1$ and $v_2 = 0$ and v_1 is directed perpendicular to immovable surface.
One is immovable and very large mass as compared to other

$v_2' = 0$; $v_1' = -v_1$

can bounces back with the same velocity.

---

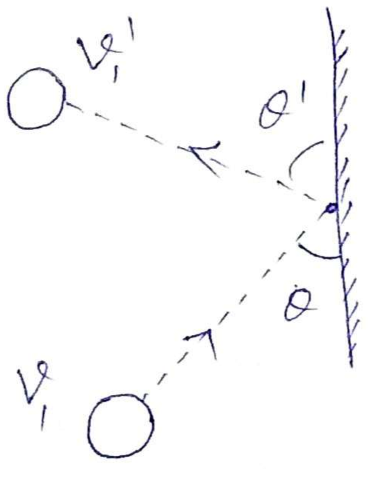
4. If a ball dropped from a height H to a ground having coefficient of restitution e, then the height of rebounce 'h' is given by $e = \sqrt{\frac{h}{H}}$

After 'n' bounces, the height of rebounce, $h_n = e^{2n} H$

---

5. When a ball is thrown against a vertical wall at an angle θ with the ~~ball,~~ wall, and it rebounds with an angle θ' with the wall as shown in fig.
The coefficient of restitution is gn by,

$$e = \frac{\tan\theta'}{\tan\theta}$$



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1. A sphere of 1kg moving at 3 m/s, collides with another sphere of weight 5kg moving in the same direction at 0.6 m/s. If the collision is perfectly elastic, find the velocity after impact

Soln:-

Apply the law of conservation of momentum

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$(1\times 3) + (5\times 0.6) = 1(v_1') + 5(v_2')$$

$$v_1' + 5v_2' = 6 \quad \text{——} \quad (1)$$

If perfectly elastic, $e = 1 = \dfrac{v_2' - v_1'}{v_1 - v_2}$

$$v_2' - v_1' = 2.4 \quad \text{——} \quad (2)$$

(1) + (2) $\Rightarrow$ $6v_2' = 8.4$

$$v_2' = 1.4 \text{ m/s } (\longrightarrow)$$

$$v_1' = -1 \text{ m/s}$$

$$v_1' = 1 \text{ m/s } (\longleftarrow)$$

2. A car weighing 5 kN is moving east with a velocity of 54 kmph and collide with a second car weighing 12 kN is moving west with a velocity of 72 kmph. If the impact is perfectly plastic, what will be the velocities of the cars.

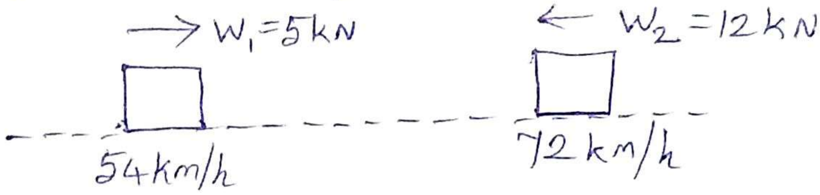
Given:-

$m_1 = \frac{5}{9.81}$ kg ; $m_2 = \frac{12}{9.81}$ kg ; $V_1 = 54$ km/hr ; $V_2 = 72$ km/

When the impact is perfectly plastic, $e = 0$

$$V_1' = V_2' = V_c \quad \text{——— (1)}$$

Apply ~~the~~ law of conservation of momentum.



$$m_1 V_1 + m_2 V_2 = m_1 V_1' + m_2 V_2'$$

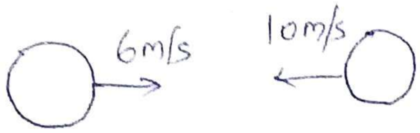
$$\frac{5}{9} \times 54 + \frac{12}{9}(-72) = \frac{5}{9} V_1' + \frac{12}{9} V_2'$$

$$= \left(\frac{5}{9} + \frac{12}{9}\right) V_c \quad [\because \text{eqn } (1)]$$

$$V_c = -34.94 \text{ km/hr. towards}$$

west with common velocity.

Direct central impact occurs between 300 N body moving to right with a velocity of 6 m/s and 150 N body moving to left with a velocity of 10 m/s. Find the velocity of each body after impact if the coefficient of restitution is 0.8.



$m_1 = \frac{300}{g}$ kg $\qquad m_2 = \frac{150}{g}$ kg

Apply law of conservation of momentum,

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$\frac{300}{g} \times 6 + \frac{150}{g}(-10) = \frac{300}{g} v_1' + \frac{150}{g} v_2'$$

$$1800 - 1500 = 300 v_1' + 150 v_2'$$

$$2 = 2v_1' + v_2' \quad \text{——— (1)}$$

$$e = \frac{v_2' - v_1'}{v_1 - v_2}$$

$$0.8 = \frac{v_2' - v_1'}{6 - (-10)}$$

$$v_2' - v_1' = 12.8 \quad \text{——— (2)}$$

solving (1) & (2)

$$v_2' = 9.2 \text{ m/s } (\rightarrow)$$

$$v_1' = -3.6 \text{ m/s}$$
$$= 3.6 \text{ m/s } (\leftarrow)$$

4. A ball is thrown against a wall with a velocity 6 m/s forming an angle is 30° horizontal. Assuming frictionless conditions and co-efficient of restitution is 0.5, determine the magnitude and direction of the ball as it rebounds from the wall.

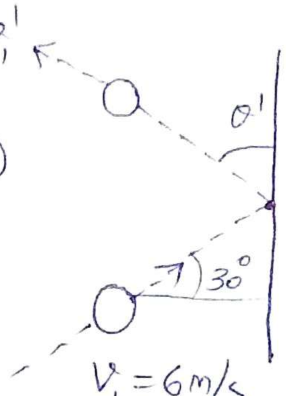


fig. ①

Soln:-

$v_{1x} = 6 \cos 30 = 5.196 m/s (\rightarrow)$

$v_{1y} = 6 \sin 30 = 3 m/s (\uparrow)$

$v_2 = 0$

collision is in the x direction;

y direction velocity is conserved.

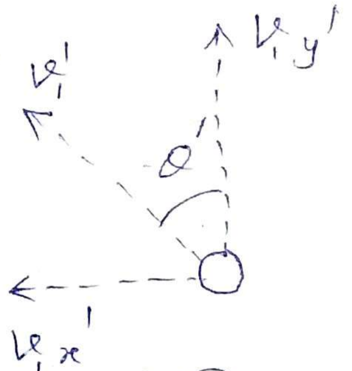


fig ②

So, $v_{1y}' = v_{1y} = 3 m/s (\uparrow)$

$$e = \frac{v_{2x}' - v_{1x}'}{v_{1x} - v_{2x}}$$

$$e = \frac{-v_{1x}'}{v_{1x}} \quad [\because v_2 = 0]$$

$$v_{1x}' = -0.5 \times 5.196$$

$$= -2.598 m/s = 2.598 (\leftarrow)$$

$$v_1' = \sqrt{(v_{1x}')^2 + (v_{1y}')^2} = \sqrt{2.598^2 + 3^2}$$

$$v_1' = 3.9686 m/s$$

from fig ②, $\tan\theta' = \frac{v_{1x}'}{v_{1y}'} \Rightarrow \theta' = \tan^{-1}\left(\frac{2.598}{3}\right)$

$$\boxed{\theta' = 40.89°}$$

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Kinetics of Particles – Translation

Newtons Second law of motion :-

The resultant force acting in the direction of motion is equal to the product of mass and the acceleration in the direction of resultant force.

$$\Sigma F = ma$$

D'Alembert's Principle:

A System of forces acting on a body in motion is in 'Dynamic Equilibrium' with the inertia force of the body

$$\Sigma F = ma$$

$$\Sigma F - ma = 0$$

'$-ma$' is called inertia force.

Where -> ΣF - be the sum of forces in the direction of motion 'N'
m - be the mass of the system in kg
a - be the acceleration of the system in the direction of motion m/s^2

1. Two blocks weighing 300N and 450 N are connected by a rope as shown in fig. With what acceleration, the heavier ~~behavious~~ block comes down, and what is the tension of the rope. Pulley is frictionless and weightless

① $\Sigma F_y = 0 \Rightarrow T - 450 - \frac{450}{g} a = 0$

Apply Newtons II law for blocks: ② $\Sigma F_y = 0 \Rightarrow T - 300 + \frac{300}{g} a = 0$

$\Sigma F = ma$

$$450 - T = \frac{450}{g} a \quad \text{——— ①}$$

$$T - 300 = \frac{300}{g} a \quad \text{——— ②}$$

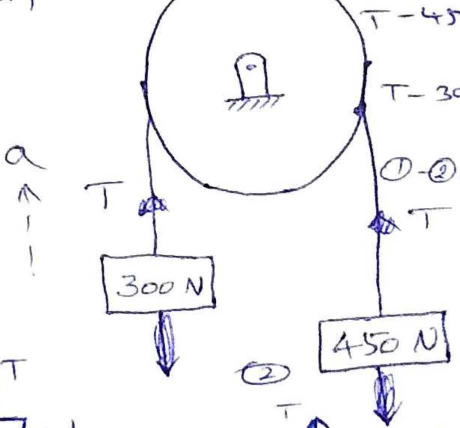
$T - 450 = \frac{450}{g} a$ ①

$T - 300 = -\frac{300}{g} a$ ②

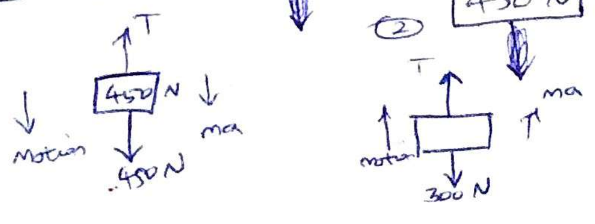
① - ② $\Rightarrow -150 = \frac{750}{g}$

① + ② $\Rightarrow$ $150 = \frac{750}{g} a$

$$a = 1.962 \ m/s^2$$



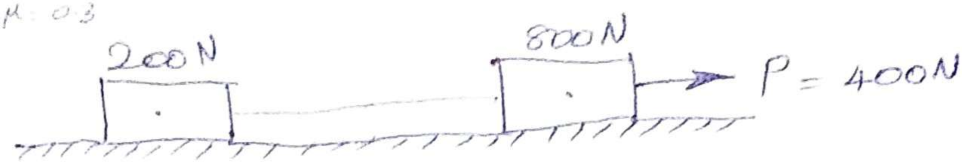
Subs in ① $\Rightarrow$ $T = 360$ N



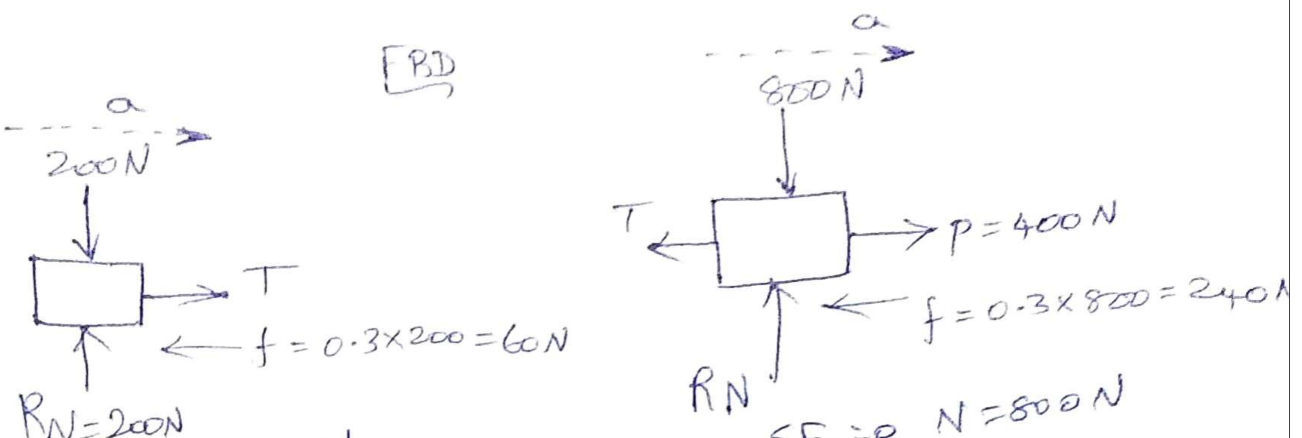
2. Two weights 800 N and 400 N are connected by a thread and they a move along a rough horizontal plane under the action of force P of 400 N applied to 800 N block, as shown in fig. Find

a) the acceleration of the weights and tension in the thread

Take $\mu = 0.3$



FBD



$\Sigma F_y = 0 \Rightarrow N = 200N$

$\Sigma F_y = 0 \quad N = 800N$

Apply Newtons II law of motion

i) for 200 N block... $T - 60 + \cancel{200} - \cancel{200} = \frac{200}{g} a$.— (1)

ii) for 800 N block $400 - T - 240 + \cancel{800} - \cancel{800} = \frac{800}{g} a$ — (2)

Solving (1) & (2), (1) + (2) $\Rightarrow 100 = \frac{1000}{g} a$

$$\boxed{a = 0.981 \ m/s^2}$$

Subs in eqn (1)

$$T = 60 + \frac{200 \times 0.981}{9.81}$$

$$\boxed{T = 80 N}$$

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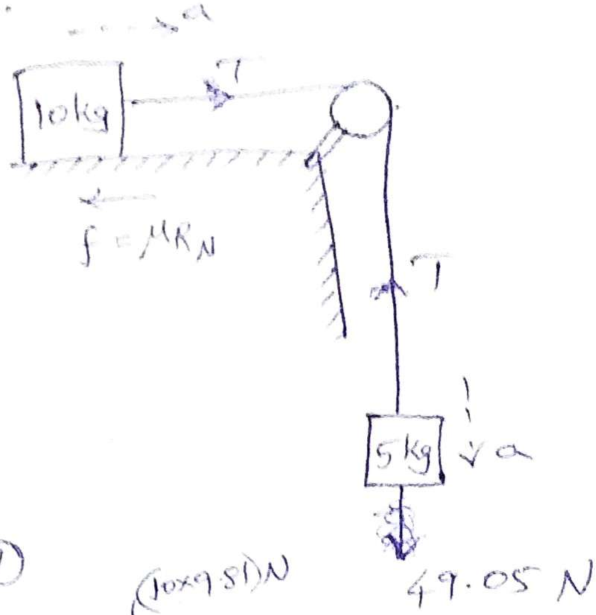
Q. Two blocks of mass 10 kg and 5 kg are connected as shown in fig. Assume $\mu = 0.25$. Find the acceleration and the tension in the string if pulley is weightless and frictionless.



Apply newtons II law

i) To 5kg block.

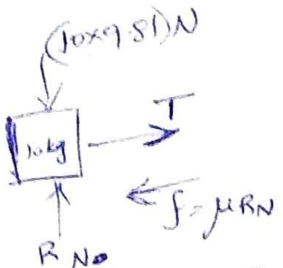
$\Sigma F = ma$

$49.05 N - T = \frac{49.05}{9.81} a$

$49.05 - T = 5a$ —— ①

ii) 10 kg block

(i)



$\Sigma F = ma$

$T - f = 10a$

$T - \mu R_N = 10a$

$T - (0.25 \times 10 \times 9.81) = 10a$

$T - 24.535 = 10a$ —— ②

$\Sigma F_y = 0 \Rightarrow R_N = 98.1$

$f = 0.25 \times 98.1$

$f = 24.535$

$\Sigma F = T - f$

$ma = 10a$

① + ② $\Rightarrow 49.05 - 24.535 = 15a$

$$\boxed{a = 1.635 \ m/s^2}$$

Subs in ① $\Rightarrow T = 24.535 + (10 \times 1.635)$

$$\boxed{T = 40.875 \ N}$$

— x —

10. In the fig shown two masses A & B are connected by a rope and pulley. The masses are released from rest. Assuming pulley are frictionless and weightless, determine (i) tension of the rope ii) Acceleration of masses A & B.

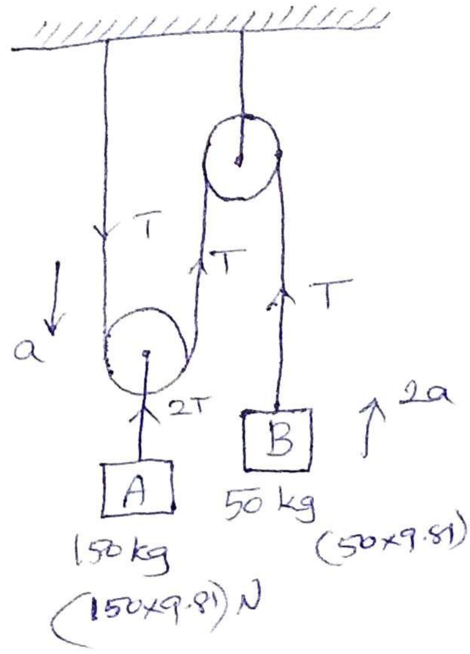
Assume the magnitude of accelerations, as

a - acceleration of 150kg downward,

2a - " 50kg upward

T is the rope of 50 kg

2T be the tension of the rope of 150 kg.



Apply Newton II law of motion

$$150g - 2T = \frac{150}{g} a \quad \text{——} \quad ①$$

$$T - 50g = \frac{50}{g}(2a) \quad \text{——} \quad ②$$

$$② \times 2 \Rightarrow \quad 2T - 100g = 200a \quad \text{——} \quad ③$$

$$-2T + 150g = 150a \quad \text{——} \quad ①$$

$$① + ③ \Rightarrow \quad 50g = 350a$$

$$a = \frac{50 \times 9.81}{350}$$

$$a = 1.4014 \ m/s^2 \qquad \therefore \boxed{a_A = 1.4014 \ m/s^2}$$

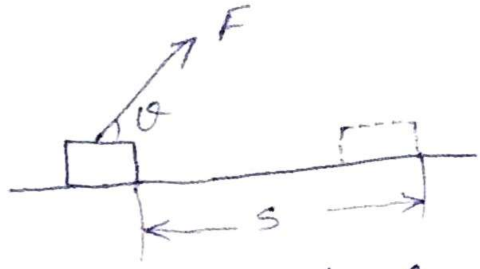
Subs 'a' value in eqn ①, $2T = 150g - 150(a)$

$$\boxed{T = 630.64 \ N}$$

$$a_B = 2(a) \qquad \therefore \boxed{a_B = 2.8028 \ m/s^2}$$

Work Energy Principle

a) Workdone by a force



Consider a force 'F' is acting on a body with an angle θ with the direction of motion. The displacement of the body is denoted by ds, then the work done by force is given by

$$dW = \int_0^s [F\cos\theta]\, ds$$

$$= \Sigma\,[\text{Force in the direction of motion}] \times \text{displacement}$$

b) Workdone by a force of Gravity:

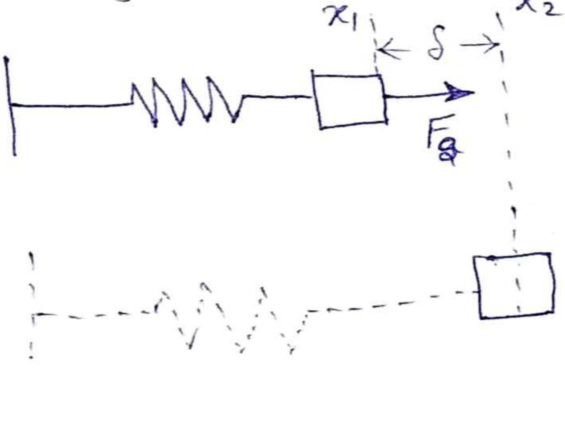
A body of mass 'm' is displaced at a height of 'h' from initial position.

$$W = \int_{Z_1}^{Z_2} -mg\, dz$$

$= -mgh$, if the displacement is upward

$= mgh$, if " downward.

c) Workdone by a Spring force



Work Energy Principle

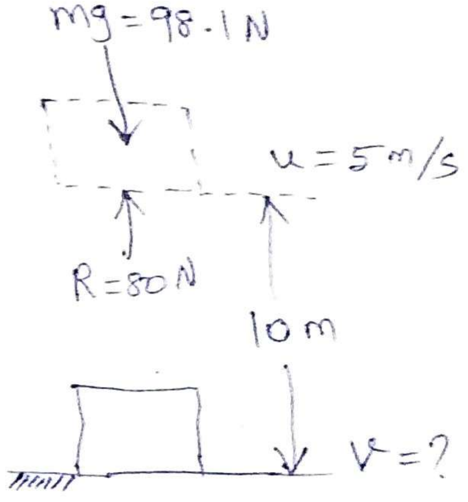
From Newton's II Law, $F = ma$

$$F \times s = \frac{1}{2} m [v^2 - u^2]$$

Workdone = Change in Kinetic Energy

This ~~is~~ equation is called work-energy principle.

1. A food packet having a mass of 10kg is dropped with an initial velocity of 5 m/s from a very high altitude, if the frictional force is 80N. Determine the velocity of food packet has fallen 10 m.

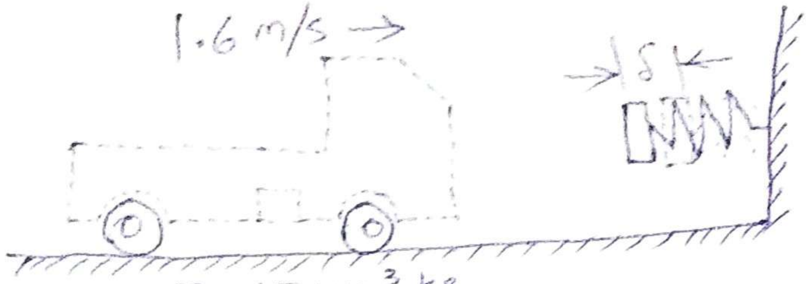


Apply with energy principle, $\Sigma F \times s = \frac{1}{2} m [v^2 - u^2]$

$$(98.1 - 80) \times 10 = \frac{1}{2} \times 10 [v^2 - 5^2]$$

$$v = 7.823 \text{ m/s}.$$

2 A truck of mass 15 tonnes travel at 1·6 m/s impacts with a spring buffer, Spring compresses 1·25 mm/kN. Find the maximum compression of Spr



$m = 15 \times 10^3$ kg

$$\text{Stiffness} = \frac{1}{1 \cdot 25} \ \text{kN/mm}$$

$$= \frac{1000}{1 \cdot 25 \times 10^{-3}} \ \text{N/m}$$

$$= 0 \cdot 8 \times 10^6 \ \text{N/m}$$

Kinetic Energy of the truck = Work done by the Spri

$$\frac{1}{2} m v^2 = \frac{1}{2} k \delta^2$$

$$\delta^2 = \frac{m v^2}{k}$$

$$= \frac{15 \times 10^3 \times (1 \cdot 6)^2}{0 \cdot 8 \times 10^6}$$

$$\delta^2 = \cancel{219}\ 0 \cdot 048$$

$$\delta = 0 \cdot 219 \ \text{m}$$

$$\boxed{\delta = 219 \ \text{mm}}$$

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Impulse Momentum Principle:

Newtons II law states that,

$$\Sigma F = ma$$

$$\Sigma F = m\frac{dv}{dt}$$

$$\Sigma F\,dt = m\,dv \quad \text{--- (1)}$$

Impulse is defined as the large amount of force acting at a short interval of time.

$$\text{Impulse } I = \int_0^t \Sigma F\,dt$$

For equation (1) Applying the limits for time 0 to t and u to v for velocity.

$$\int_0^t \Sigma F\,dt = m\int_u^v dv$$

$$= m[v-u]$$

Impulse $\Sigma F \times t = mv - mu$ = Change of momentum.

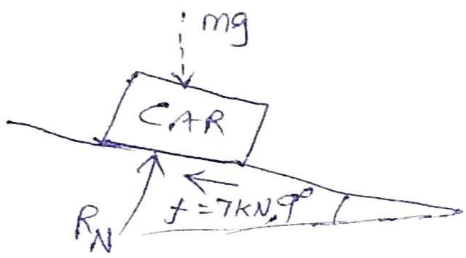
∴ Net force in the direction of motion X time = Change of momentum

Note:- When net force acting on the body is zero; Initial momentum = final momentum.

This is called law of conservation of momentum.

1. A 2500 kg car is driven down a 9° inclined plane at a speed of 100kmph. When brakes are applied causing a constant total braking force is 7 kN, determine the time required to stop the car.

Given:-



$u = \frac{100}{3.6} = 27.78 \text{ m/s}$

$v = 0$

$f = 7000 \text{ N}$

Soln:-

Apply Impulse-Momentum principle:

$$\Sigma F \times t = m[v-u]$$

$$-7000 \times t = 2500[0-27.78]$$

$$t = 9.9214 \text{ sec.}$$

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2. A 10gm bullet has a velocity of 2km/s as it enters a fixed block of wood. It comes to rest in 0.002 seconds after entering the block. Determine average resistance force that acted on the bullet and the distance penetrated by it.

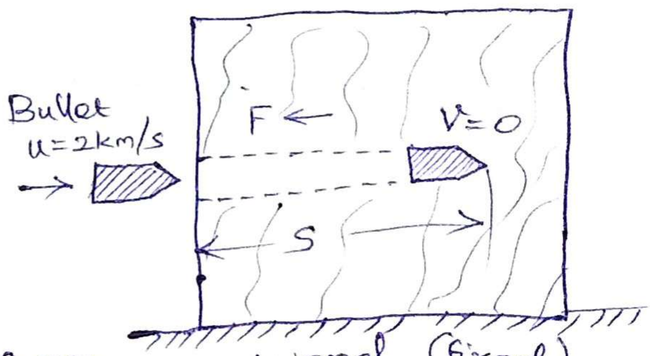
Given:-

$u = 2000 \text{ m/s}$; $v = 0$

$m = 10 \text{ gm}$

$m = 10 \times 10^{-3} \text{ kg}$

$t = 0.002 \text{ sec.}$



Let 'F' be the avg. resistance force

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